MU120 Unit 2

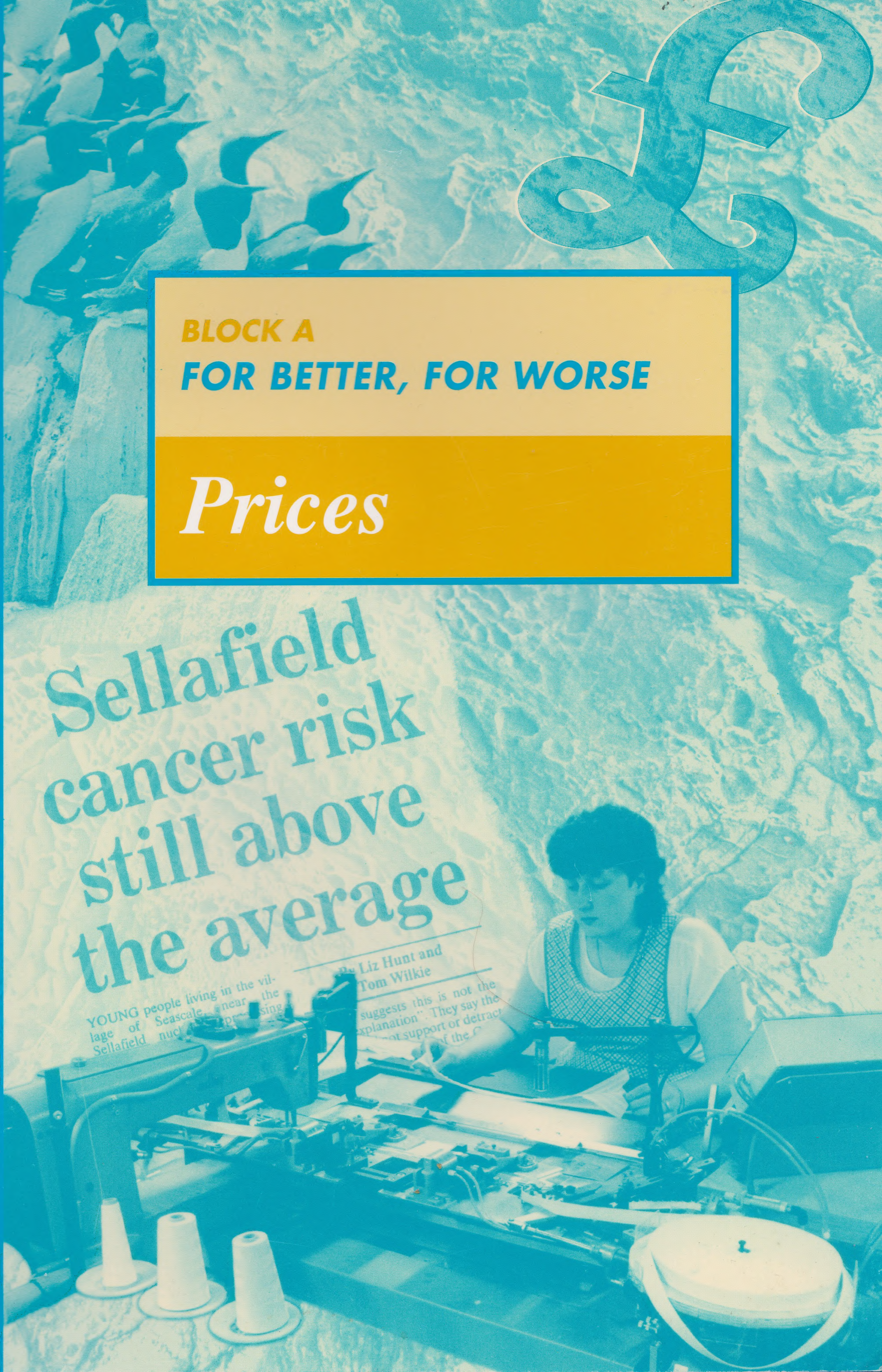
The Open University

Mathematics and Computing
A first level multidisciplinary course

Open Mathematics

BLOCK A
FOR BETTER, FOR WORSE

Prices



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Contents

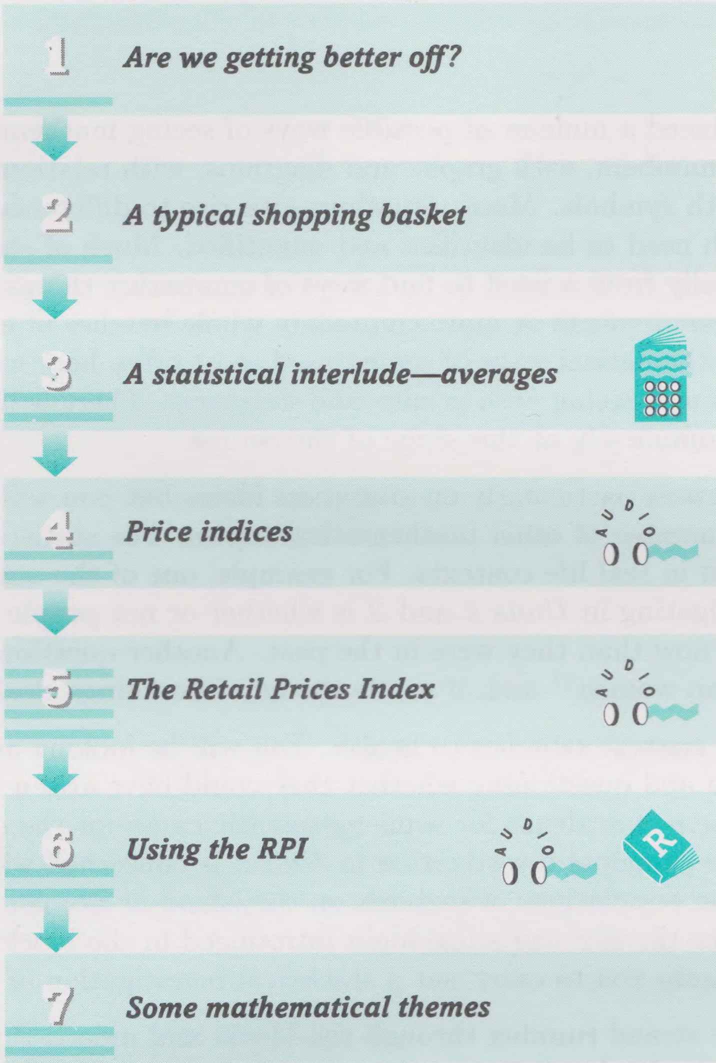
Study guide

Unlike *Unit 1*, this unit is larger than average and has been allocated a longer study time. It consists of seven sections, but they are not of equal length. You should find that both Sections 1 and 2 may comfortably be studied in one two-hour session, whereas the remaining sections will probably occupy about the same time each (although Section 5 is likely to need more and Section 6 slightly less time). Section 3 involves working through two sections of the *Calculator Book*: here you will begin to explore some of the statistical facilities of your calculator. Section 4 is a central section—the ideas of price ratios and price indices are developed here—and it contains an audiotape sequence. These ideas are applied in Sections 5 and 6, where the price index (the Retail Prices Index) used by the British Government to measure changes in prices is discussed. Both Section 5 and Section 6 also contain an audiotape sequence, while Section 6 additionally contains some activities based on a reader article. In the final section, you are invited to step back from the details of price ratios and price indices to consider some of the mathematical ideas that have been used and developed in the earlier sections of the unit.

You will need the main text and your calculator for all the study sessions. There is no videotape material associated with this unit. The television programme *Wood, Brass and Baboon Bones'* describes how data have been recorded, represented and manipulated using human-constructed devices. Although the study plan provides only a rough guide to the time that you will need to spend on each section (recall that each horizontal bar represents approximately one hour), it should nevertheless help you to schedule your study of the unit.

Associated with your learning of this unit is a Handbook activity sheet and also some Learning File sheets. The Handbook activity sheet is not restricted to just one activity—it is designed to be used at different times. as you work through the unit. Activities indicate when you may find it appropriate to use it.

Before you begin work on Section 1, you are encouraged to plan your work on the unit as a whole. Take a look at the assignment booklet to see what you need to do and when: be sure to build in enough time to do the TMA questions. You may wish to revisit the notes in *Preparing for Open Mathematics* on preparing a TMA.



Summary of sections and other course components needed for *Unit 2*

Introduction to Block A

Unit 1 introduced a number of possible ways of seeing mathematically: looking with numbers, with graphs and diagrams, with relationships, with generality, with symbols. Many situations give rise to differences or changes which need to be identified and quantified. Much of statistics develops initially from a need to find ways of comparing things, either individual measurements or more commonly whole batches of numerical data. The mathematical ways of seeing used most often here will be seeing with numbers and seeing with graphs and diagrams. There is little work on looking symbolically at this stage of the course.

This block focuses particularly on statistical ideas, but you will also be developing a number of other mathematical topics. The statistical themes are introduced in real life contexts. For example, one of the questions you will be investigating in *Units 2* and *3* is whether or not people in the UK are better off now than they were in the past. Another question is 'Do men earn more than women?' and, if so, 'Is the gap in earnings closing?'

In *Unit 4*, the context switches to health. You will be looking at patterns in health data and questioning whether they could have arisen by chance or whether there is evidence for some systematic cause for the effects observed. The principal investigation in *Unit 5* is concerned with monitoring the populations of seabirds on the island of Skomer. This unit brings together the key statistical ideas introduced in the block and is intended to equip you to carry out a statistical investigation of your own.

An important strand running through the block, and indeed through the whole course, is the development of your critical awareness when processing information. Working on making sense of various reader articles will contribute to this.

Finally, the calculator is a central component in your study of this course. In this block, you will be exploring its use principally through the statistical facilities it provides. Rather than seeing it solely as a calculating device, remember it is also a learning aid.

Introduction

Are we getting better off? Politicians and journalists often make sweeping claims about whether or not 'we' are getting better off.

As this unit is being written, some UK journalists are writing of the 30/30/40 society. These numbers refer to percentages of the general population: those who are disadvantaged (including the unemployed), those who are marginalized and insecure (in relation to the labour market), and the privileged (whose market power has increased).

- Who is this 'we' of whom they speak?
- On what do they base these claims?
- What does being better off mean to you?
- How would you go about assessing how well-off you are?

In attempting to resolve some of these questions, a number of important mathematical and statistical ideas arise. For example, these questions suggest a need to measure something. While it may be pretty clear how to measure say height or temperature, it is by no means clear how to set about measuring something rather vague like 'well-offness'. This unit begins by looking at an everyday measure which affects how well-off many people feel—the price of a loaf of bread. You will be asked to think about how comparisons over time might be made. The unit then focuses on prices: how to measure them and how to measure price changes over time.

As you work through the unit, you will be using a number of mathematical skills and acquiring some new ones; and you will be meeting some mathematical and statistical ideas: data and various ways they can be presented, percentages, averages including weighted means, price ratios and index numbers.

You will also be building on ideas introduced in *Unit 1*: particularly looking mathematically with numbers, and with tables and graphs. You are encouraged to think about how such structural images are used, and look at communicating and improving your own learning and performance. You will also be introduced to statistical problem solving.

Now that you have organized your Learning File, it is important that you get into the habit of using it to record your notes. Sometimes, specific suggestions of things to record will be made, but these are only a minimum of the possible entries, and the more opportunities you create for yourself the better. The Learning File activities and Handbook activities all have printed response sheets.

The Learning File should become your personal record of your learning throughout the course. For it to be most useful to you, it is important that you keep it organized systematically, so that you can easily find anything you want to refer to. You may like to include your assignments and other notes. But it is for you to decide how to record other activities and exercises. Continue thinking about this while you are working on the unit.

1 Are we getting better off?

Aims The main aim of this section is to introduce some ideas about making valid comparisons and to focus on ways of extracting information from tables and graphs. ◇

1.1 Using your loaf

> Cade: There shall be in England seven halfpenny loaves sold for a penny; the three-hooped pot shall have ten hoops; and I will make it felony to drink small beer. All the realm shall be in common, and in Cheapside shall my palfrey go to grass. ... there shall be no money; all shall eat and drink on my score; and I will apparel them all in one livery, that they may agree like brothers, and worship me their lord.
>
> (*William Shakespeare, Henry VI, Part 2*)

In this quotation, the character Cade anticipates the good times that are sure to follow after the revolution. The notion that they shall 'worship me their lord', however, raises some doubts about just how egalitarian this Utopian society might really be!

The notion of the 'halfpenny loaf' is interesting, as is the question of whether most people in Shakespeare's Elizabethan England were able to afford it. It raises the general question of how well-off different people actually were in the late sixteenth century when the play *Henry VI, Part 2* was written, and whether, in material terms, people are much better off now. This is the central question which drives this unit and the next. It provides a context for exploring ideas such as the cost of living, the standard of living, and measuring inflation using the Retail Prices Index (RPI).

To start with, you will be asked to engage in a problem-solving process. One way to learn is by solving problems. You may remember from *Unit 1* examples of how people felt they learned—often, it is through engaging with a problem or situation they needed to solve or were curious about or fascinated by. In this case, you will work through a series of stages to try to answer the question 'Are people getting better off?' The first stage here is to try to specify the problem more precisely.

Activity 1 Using your loaf

Comments on Activities begin on page 77.

Can you think of a way of using the loaf of bread as a very rough measure of how the standard of living has changed between the late sixteenth century and today? What information would you need?

Take a minute or two to write down your ideas before reading on.

Clearly, the price of a loaf of bread has greatly increased since 1594 (the year that the play was written). However, prices and earnings have both increased, so simply looking at prices alone will not provide a useful basis of comparison. One simple measure is the percentage of someone's daily or weekly earnings required to buy a loaf of bread. Making such a comparison requires further pieces of data: the price of a loaf of bread today, and *typical* earnings in 1594 and today. You may well have suggested something different.

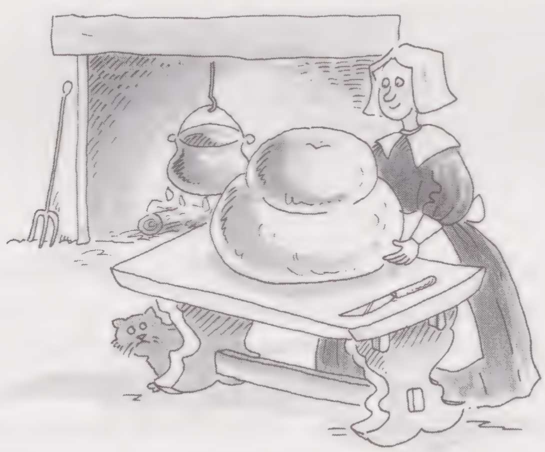
With this formulation of the problem, there is a need to collect some appropriate data. Note the use of the word *data*. In this context, *data* means information, usually appearing in the form of numbers. Data is a plural word (the singular, *a datum*, refers to a single fact or figure) and data are the building bricks of any statistical investigation.

In everyday usage, the word 'data' is often used in the singular, but MU120 treats the word as a plural noun.

Activity 2 Collecting the data

How would you go about finding out the information necessary for the comparison of the price of a loaf of bread and typical earnings between 1594 and today? What problems of measurement and comparison do you foresee? Write down your ideas before reading on.

First, what about the price of a loaf of bread today? If you look in almost any shop selling bread, it is likely that you will be faced with a bewildering range of types of bread—white, brown, granary, wheaten, baps, batches, French sticks, and so on. Bread also comes sliced or unsliced, and in a variety of sizes—large, medium and small. Not surprisingly, each of these many options comes at a different price. So, what *is* the price of a loaf of bread? The comparison perhaps requires choosing a loaf that corresponds most closely to the sort of loaf that people were buying four hundred years ago. Did they go in for baking giant loaves that would see a family of twelve through the entire week or were Elizabethan loaves gobbled up in a single bite?



Who knows, without a lot of further investigation? Suddenly this is getting much harder! This is rather specialized knowledge that most people are unlikely to have. However, exact figures are not necessarily appropriate here, so let us choose the most popular large loaf that most people these days seem to put in their shopping trolleys: a large white sliced loaf.In 1994, this was priced at 56p in a local UK supermarket.

Next, what was a typical wage in 1594? This required a library search, and, with a little help from the computer, a book called *The Population History of England 1541–1871* by E.A. Wrigley and R.S. Schofield came to light. It revealed that the daily wage of building craftsmen in southern England between 1580 and 1629 was 12 old pence (there were 240 old pence in a pound).

Finally, what could stand for a typical wage today? For reasons of comparability, it seems to make sense to choose a modern UK manual worker (although there is a case for choosing a skilled worker instead) and a figure of roughly £250 per week seems a reasonable 'estimate' in 1994.

Now that all the required data are collected, the next stage in solving the problem is to analyse them. This will involve calculating one of the measures suggested, namely the percentage of daily earnings required to buy a loaf of bread both in 1594 and in 1994.

Extracting data from text can be confusing: you need to be able to sort out what is valid; what is misleading; and what is simply irrelevant. Tables can often help to clarify matters.

Table 1 Summary of wages and the price of a loaf, 1594 and 1994

	Then (1594)	Now (1994)
Price of a loaf	$\frac{1}{2}$ or 0.5 old penny	56 new pence
Typical wage	12 old pence per day	£250 per week

Activity 3 Taking in the table

Pause for a few moments and reflect on what this table of data means to you.

◇ Write down briefly in your own words what the table is telling you.

◇ Make a brief note of anything that puzzles you.

When constructing a table of this kind, one of the first checks to make is to ensure that the units are compatible. Clearly, one old penny in 1594 was worth much more in terms of what it could buy than one new penny today. However, since the necessary calculations will run down the columns of the table, there are no direct comparisons between old money and new money, so the relative worth of the coinage will not cause a problem.

One difficulty still to be dealt with is the basic wage unit. This is quoted at a daily rate for 1594 and at a weekly rate for 'now'. In order to be able to make proper comparisons, the same period of time should be used. So one of the rates must be converted. It does not really matter which, so, to keep the numbers small, why not convert this weekly rate to a daily rate?

However, the best procedure for this calculation is not obvious. Although there are seven days in a week, how many *working* days are there in a working week now and how many were there in Elizabethan times? It could be five, six or seven or something between. On balance, because people have to eat seven days a week, probably divide by seven. However, it is worth pausing to observe that this is one of many situations where ambiguity and uncertainty crop up in statistical work. In general, problems and investigations which involve these sorts of subjective judgements are not so much 'solved' as 'resolved'.

A weekly wage of £250 works out at £250 ÷ 7 which is £36 or 3600 new pence per day (correct to two significant figures).

Table 2 Summary of wages and the price of a loaf, 1594 and now—daily rates

	Then (1594)	Now (1994)
Price of a loaf	$\frac{1}{2}$ or 0.5 old pence	56 new pence
Typical wage per day	12 old pence	3600 new pence

Note that Table 2 shows the same information as Table 1 but with *daily* rates quoted in *pence* in each case. Now express the cost of a loaf as a percentage of a typical daily wage.

Percentages were covered in the preparatory materials and *Unit 1*. If you are unsure about them, it may be a good idea to revise them now.

The cost of a loaf in 1594 as a percentage of a typical daily wage is:

$$\frac{\frac{1}{2}}{12} \times 100\% = \frac{0.5}{12} \times 100\% \simeq 4.2\%.$$

Historical note

Notice the use of the symbol $\simeq$, which means 'is approximately equal to'. It is similar to the equals sign. But if you remember Robert Recorde's comment mentioned on the preparatory audiotape 'Communicating Mathematically'—'Noe two thynges could be moare equalle'—the wavy line here suggests 'close, but not quite exact'. It serves as a reminder that rounding or some other means of approximation has been used.

Activity 4 Today's figures

Calculate the cost of a loaf today as a percentage of the typical current daily wage. Compare the result for today with the corresponding figure for 1594. Do the results suggest people are better off today than four hundred years ago?

The percentage figure for 1594 at 4.2% is more than twice the corresponding figure for 1994, 1.6%. In other words, as a percentage of earnings, the cost of a loaf of bread has dropped to less than half *in real terms* over the past four hundred years. This seems to suggest that people have got better off—more than twice as well-off, in fact.

Having collected and analysed the data, the next task in the problem-solving process is to interpret the results. Comparison over time based on the cost of a simple item such as a loaf of bread is relatively *straightforward*. But is it *appropriate* as a way of measuring how well-off people are? It is possible that the price of bread is highly untypical and so some other goods should be used. But what else might you choose? Clearly, to choose items whose existence are exclusive to recent times, such as cars, computers and electricity bills, is inappropriate. Yet to exclude these items will have the effect of denying their contribution to how well-off or otherwise you may feel today. For similar reasons, the price of quill pens, ox-drawn ploughs and hunting spears should probably be excluded.

Even things which existed both then and now, like houses, clothing and heating, are so different in nature as to be, essentially, different items. Furthermore, it is quite possible that many of the goods and services that made living in Elizabethan times bearable, if not pleasurable (the acquisition of fresh vegetables, child care, and so on), were not bought with cash but 'paid for' in kind or by favour or grown for personal consumption. The tax system has also altered considerably over the period, as has the range of services provided in a subsidized or free form by the state (health, education, social welfare, ...). All of these services contribute hugely to the quality of life in the twentieth century and were largely absent in this form four hundred years ago.

A final complication is that the data on which the previous analysis was based were restricted to a particular group within Elizabethan society. The workers were first men, second building craftsmen, and third those building craftsmen who resided in southern England. This is a very particular group and it would be dangerous to draw universal conclusions covering women, children, the elderly, peasants, landowners and town-dwellers of the time. Today, earnings vary enormously both within and between occupations and similar inequalities certainly operated in the late sixteenth century.

In summary, then, this investigation which started out with a deceptively simple question has become something of a puzzle! The 'hard facts' required have proved to be slippery indeed. There is a lot of information that we simply do not know and have had to make sensible guesses about. It should be stressed that this is a more common state of affairs than is popularly realized! How much of the quantitative information supplied by government, advertisers and the media might have been constructed in this way? Try to develop a habit of considering whether facts presented to you are sensible and consistent, and checking other people's sources of information.

An important thread that has run through this subsection has been problem solving. You have started to learn about some ideas in statistics through addressing this problem. A number of key stages in the solving of a problem have been employed. They concern clarifying the problem, the collection and analysis of the data, and their interpretation. These will be mentioned from time to time in your work over the first three units of this block, but will be drawn out and discussed in some detail in *Unit 5*.

1.2 The price of a loaf these days

The investigation so far illustrates just how difficult it can be to make a fair comparison of prices, particularly over a very long period of time during which the world has changed out of all recognition. In this section, the central question is still 'Are people getting better off?' but, in order to make the task more straightforward, we chose a far narrower time interval, looking just at the period from 1980 to 1994.

Activity 5 Collecting the data

How might you use the 'price of bread' measure as a way of investigating whether or not people got better off over this period?

In particular, think about:

(a) the data you might wish to collect;

(b) where you might look for the information you need.

The data used in Activity 4 were taken from just two points in time. An alternative approach is to collect a series of values at regular intervals over the period in question. Suitable data indicating the price of bread in the UK can be found by looking through back copies of a monthly publication called the *Employment Gazette*. Average prices for a selection of fairly standard goods are included each month. However, extracting the information that you need is usually not straightforward and you would have to be fairly selective. Here, for example, is the entry in the *Employment Gazette* for April 1993 under the heading 'Bread'.

Table 3 Average prices of bread on 9 February 1993

Type of loaf	Number of quotations	Average price (pence)	Price range containing 80% of quotations (pence)
800 g white loaf, sliced	349	55	39–76
800 g white loaf, unwrapped	338	73	67–82
400 g white loaf, unsliced	348	48	44–53
400 g brown loaf, sliced	339	51	41–56
800 g brown loaf, unsliced	331	77	73–84

Source: *Employment Gazette, April 1993, page S61*

First of all, you need to sort out what all these figures mean in order to select the information that is required.

Activity 6 *Getting a grip on your table*

(a) The information in Table 3 was gathered from a survey of shops. Roughly how many shops were surveyed? (Note, it is *not* the total of the numbers in the 'Number of quotations' column.)

(b) On the basis of the average prices quoted here, what was the cheapest type of *large* loaf?

(c) Using the information in the final column of the table and your common sense, estimate how much you might have to pay for one of the cheapest of the small brown loaves.

(d) Which type of loaf showed the greatest variation in price? Why do you think this is?

(e) How does presenting data in the form of a table help you to make sense of it?

As you continue to work on this block, keep a record in your Learning File of how tables help you to process data.

Consider how one of the prices, the 55p average price for a large white sliced loaf has changed over the period. Table 4 below gives the corresponding average prices for this item in February over the period in question.

Table 4 Average February prices of a large white sliced loaf from 1980 to 1994

Year	'80	'81	'82	'83	'84	'85	'86	'87	'88	'89	'90	'91	'92
Price (p)	32	36	37	38	39	40	42	43	45	49	49	54	54

Year	'93	'94
Price (p)	55	51

Source: *Various issues of the Employment Gazette, 1980 to 1994*

As with many tables, it is not easy to see any clear pattern from these figures alone. It is often helpful to re-present numerical information using a different form: a graph. This is shown in Figure 1.

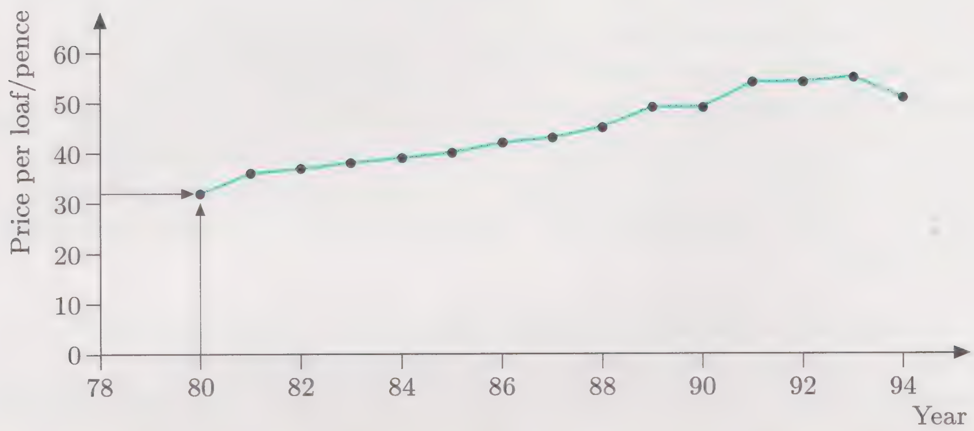


Figure 1 Graph of average February prices of a large white sliced loaf from 1980 to 1994

The two arrows on the graph point to the first plotted point, which corresponds to the first pair of values from Table 4, namely the year 1980 and the price 32 pence. The point is positioned by lining up the value 80 on the 'Year' axis and the value 32 on the 'Price per loaf/pence' axis. Each of the other points is plotted using the same principle.

Activity 7 *Graphing the data to see patterns*

(a) Check that you understand how the points have been plotted. For example, the second point on the graph should correspond to the second pair of values from Table 4, namely the year 1981 and the price 36 pence.

(b) The adjacent points on this graph have been joined with straight lines. What is the meaning of these lines and why is this procedure appropriate in this example?

(c) Some parts of the graph show a steeper slope than others. Identify intervals of time where the graph is particularly steep and explain what this signifies.

(d) Note down any features that you observe in the position of the points. What do they tell you that might be of interest about how bread prices have changed in general over time?

Activity 8 *Presenting data*

In Table 4, and in Figure 1, the same information is presented in two different ways. There are many occasions when going backwards and forwards between a table and a diagram is important, because different ways of representing data stress and ignore different things. Which did you find easier to interpret, the graph or the table? Which seemed more helpful and which more appropriate to the problem?

Recall the discussion from *Unit 1* on stressing and ignoring.

Now think more generally about tables and graphs that you have seen in the world of represented data about you. Try to describe how using different forms of images enables you to make sense of the data represented. Find examples where you can. Concentrate on whether there are any advantages in portraying information by means of a graph, as opposed to a table (or even in words). Does your answer depend on the purpose for which the information is required, or on any other particular circumstances?

There is a printed response sheet for this activity. You may wish to continue to add your ideas to it as you work through the unit

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As you work through this unit, you will find other kinds of diagram being used to convey information. Use your printed response sheet to record other ways of presenting data that you meet and make notes about usage/advantages, and so on. We come back to this issue at the end of the unit.

It should be clear (and unsurprising) from Figure 1 that bread prices rose over this fifteen-year period. However, the rate of increase has not been entirely steady: for example, over the year between February 1980 and February 1981, the graph shows a fairly steep increase, whereas over the next four years the rate of increase is much more modest. After February 1991, there seems to be a levelling off of prices and the graph actually fell substantially over the final year under consideration. Now return to the data in Table 4 to check some of the details and their implications more carefully.

Between February 1981 and February 1982 the price increase was 1p. Similarly, between February 1992 and February 1993 the price increase was again 1p. You might say that the first price increase was the same as the second one. While this is a correct statement, it is rather misleading. A rise of, say, 1p on a loaf costing 36p is actually of greater importance than a similar rise on a loaf costing 54p. To take a more extreme case, an increase of 10p in the price of a newspaper is far more important than an increase of 10p in the price of a new car. A more informative way of describing price rises is to express them as percentages of the original price of the item in question. So if a newspaper costing 50p went up in price by 10p, this would represent an increase of one-fifth, or 20% of its original price.

To calculate the percentage increase in the average bread price from 1981 to 1982, use your calculator to carry out the following calculation.

> Divide the price increase, 1p, by the original price, 36p, then multiply by 100 to turn the answer into a percentage.

In scientific and technical work, long strings of digits are usually separated by spaces to make them easier to read; for example, 2.777 777 778.

Thus:

$\frac{1}{36} \times 100$ gives the result 2.777777778,

or 2.8% rounded to one decimal place.

Activity 9 Over to you

(a) Using the same approach as for the calculation above, use the data from Table 4 to work out the percentage increase in the average bread price between February 1992 and February 1993.

(b) How would you now modify the earlier statement that 'the first price increase was the same as the second one'?

(c) Calculate the percentage increase in the average price of a large white sliced loaf between February 1980 and February 1993.

How have wage rates changed over the same period? Note that when this section of the course was written, the only available figures for earnings were up to 1993. Have a look at Table 5, below.

These figures refer to the average (mean) gross full-time earnings including overtime, for those men whose pay was not affected by absence. The mean is discussed in Section 3 of this unit.

Table 5 Average UK male earnings (weekly) for all industries and services from 1980 to 1993

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988
Rate (£)	111.7	121.9	133.8	143.6	152.7	163.6	174.4	185.5	200.6

Year	1989	1990	1991	1992	1993
Rate (£)	217.8	237.2	253.1	268.3	274.3

Source: *The Family Expenditure Survey, 1980–1993*

Activity 10 Using percentages to make comparisons

(a) Calculate the percentage increase in average male weekly earnings between 1980 and 1993.

(b) Compare your answer for part (a) with your answer for part (c) of Activity 9. What conclusions can you draw by comparing these two values?

Overall, the results of the various calculations in the previous two activities seem to suggest that, while bread prices and earnings both rose throughout the period in question, male weekly earnings rose more in percentage terms than bread prices. So does this prove 'we were all better off in 1993 than in 1980'?

There are several reasons that such a conclusion does not necessarily follow.

First, the earnings figures refer only to men in the UK.

Second, the earnings figures are averaged out over a wide range of jobs. Without further research, you know nothing about particular groups of workers—agricultural workers, construction workers, and so on—some may be better off, others may be worse off.

Third, not everyone was in employment. In fact, over the period in question, the percentage of the workforce who were unemployed fluctuated between about 7% and 12%. So, for the three million unemployed people in 1993 (or roughly one in twelve of the work force), this average rate of pay of over £270 per week would be a complete irrelevance.

A fourth source of doubt is that changes in bread prices alone are a poor measure of how prices have changed as a whole. Bread purchases represent only a small fraction of typical weekly shopping baskets, and so we really need to take account of a much wider range of goods. How to choose and analyse a suitable basket of goods is the central issue of the next section.

Before you go on to the next section, this would be a good point at which to check that you have made a record of each activity you completed in this section.

The main aim of Section 1 has been to pose the central question of *Units 2* and *3*, namely how to assess whether people are materially better off today than in the past. Data were collected and analysed and some interpretations were made of the results of this analysis. At this stage, one conclusion seems to be that, in general terms, we have become better off. However, this conclusion is only tentative, for you have seen how a superficial quantitative approach can be misleading. There are more formal and more accurate ways of investigating this central question: in particular, two crucial measures—of prices and of earnings. The rest of this unit concentrates on prices; the next unit looks at earnings.

Outcomes

After studying this section, you should be able to:

- ◇ consider and decide what data are required to make comparisons of prices (Activities 1, 2, 4, 5);
- ◇ calculate a percentage and a percentage increase (Activities 4, 9 and 10);
- ◇ read and interpret data from a table (Activities 3, 6);
- ◇ read and interpret information from a graph (Activity 7);
- ◇ make some comments about the differences between these two ways of representing data (a table and a graph) (Activity 8).

2 A typical shopping basket

Aims The main aim of this section is to discuss how best to measure average price increases. ◇

In this section, we will try to identify a typical basket of goods and use these goods to analyse price changes over time. However, what is meant by 'typical'?

Think back to the last time you went shopping—if you can find one, dig out a recent shopping receipt. Have a look at the sorts of item that are listed on it. No doubt there are various different items: like washing powder, toilet rolls, rice, potatoes, milk, lentils, sausages, electric light bulbs, ... But what about those sausages? They certainly will not ever be in your basket if you follow a vegetarian diet. And the electric light bulbs that you have just stocked up on are unlikely to be in your shopping basket next week, whereas the potatoes and milk may well be. Of course, the items that you did not buy this week—maybe a new toothbrush or a jar of curry powder—will not be counted at all.

To monitor price changes in a way that takes account of all goods, it is not enough merely to consider those items that you buy in a supermarket in any particular week or even all the items that you might ever buy there. Suppose, for example, that in your prices survey you had bought a new bicycle. This would ensure that bicycle expenditures would be included. But the cost of the bicycle is likely to outweigh everything else in your 'shopping basket' and would give bicycle expenditure undue importance in your weekly budget.

So, merely taking one person's basket for one particular week is actually likely not to be typical of anything very much at all. In order to make a

thorough job of finding a 'typical' shopping basket, we really need to take a sample of different people and follow their purchases over a period of weeks or months. This more thorough approach is actually what is done by the government organization which monitors price changes, and the procedure is described in more detail in Section 5 of this unit.

However, for reasons of simplicity, we focus in this section on a small 'shopping basket' containing just five items of food bought by a very large proportion of households in the UK. Do not worry too much at this point about the many omissions or the fact that for some people this basket of goods is actually rather unrepresentative of their purchases. It is simply being used to illustrate some of the main ideas involved in measuring and comparing price changes. The five food items chosen here are bread, milk, eggs, potatoes and sugar. Your five may be very different; you will examine your own expenditure patterns later in the unit.

Here is a first attempt at calculating some sort of average price increase over the fifteen-year period from 1980 to 1994. The data are given in Table 6 and have been quoted to the nearest penny.

Table 6 1980 and 1994 prices for a small basket of goods

Item	July 1980 price	July 1994 price	Increase
Large loaf (white)	34p	50p	16p
Milk (1 pint, ordinary)	17p	36p	19p
Eggs (1 dozen, size 2)	72p	136p	64p
Potatoes (1 kg, loose)	20p	42p	22p
Sugar (1 kg)	36p	67p	31p
Total			152p

Average (mean) price rise of all the goods is $\dfrac{152}{5} = 30.4\text{p}$.

Activity 11 Calculating an average price increase

Look at the calculation above. Do you feel the answer is useful as a measure of an average price increase? If not, then explain why not.

You may or may not have noticed that this calculation is rather unhelpful and the result of '30.4p' is consequently a pretty meaningless figure. To demonstrate this, suppose that the potatoes happened to be bought as a 50 kg sack, rather than as one kilogram. Under these circumstances, the calculation would be as shown in Table 7.

Table 7 1980 and 1994 prices for a basket containing a large sack of potatoes

Item	July 1980 price	July 1994 price	Increase
Large loaf (white)	34p	50p	16p
Milk (1 pint, ordinary)	17p	36p	19p
Eggs (1 dozen, size 2)	72p	136p	64p
Potatoes (50 kg, loose)	1000p	2100p	1100p
Sugar (1 kg)	36p	67p	31p
Total			1230p

Average (mean) price rise is $\dfrac{£12.30}{5} = £2.46$.

So, simply altering the units in which the potatoes have been measured has made a dramatic difference to the overall average. Indeed, potatoes have now moved from being the third most significant item in the shopping basket (contributing only 22p to the total price increase) to the most significant (contributing £11 to the total). The same problem applies to all the other items. Why choose the unit of milk as one pint? You could have chosen one litre, or four pints, or anything at all. Similarly, there is nothing special about choosing a dozen eggs. The amount could have been half a dozen or 144 or something else.

The quantity of each item that is chosen is crucially important to this calculation, as it determines the 'weighting' of that item in the overall average.

▶ What might be a fairer way of choosing a suitable weighting for an item?

One possibility might be to dispense with price increases expressed in pounds and pence and work only with percentage increases for each item.

A second attempt at calculating a measure of the average price increase over the fifteen-year period from 1980 to 1994 is shown in Table 8. The percentage increases in the table are given to the nearest whole number.

Table 8 Calculating percentage price increases

Item	July 1980 price	July 1994 price	Increase		% Increase
Large loaf (white)	34p	50p	16p	$\frac{16}{34} \times 100$	$\simeq$ 47%
Milk (1 pint, ordinary)	17p	36p	19p	$\frac{19}{17} \times 100$	$\simeq$ 112%
Eggs (1 dozen, size 2)	72p	136p	64p	$\frac{64}{72} \times 100$	$\simeq$ 89%
Potatoes (1 kg, loose)	20p	42p	22p	$\frac{22}{20} \times 100$	$\simeq$ 110%
Sugar (1 kg)	36p	67p	31p	$\frac{31}{36} \times 100$	$\simeq$ 86%
Total					444%

Average percentage price increase is $\dfrac{444}{5} = 88.8\%$.

Activity 12 Calculating an average price increase measure using percentages

Look at the calculation above. How useful do you feel this answer is as a measure of an average percentage price increase?

This second attempt at calculating an average price increase is an improvement in one respect. Dealing only with percentages has solved the problem of having to decide in which units to measure each item—the percentage price rise of potatoes is 110%, regardless of whether they were bought in amounts of 1 kg or 50 kg. However, the final stage of the calculation, which involved adding the five percentages together and dividing by five, has resulted in giving each item the same emphasis. As a measure of how these price changes affect the standard of living, this is not very sensible. It is unlikely that each of these items makes an equal impact on someone's budget. For example, on any given day you may consume one pint of milk but not use any sugar at all.

In everyday language, people often talk about emphasis in terms of 'weighting' one thing more heavily than another. This physical image is quite helpful to bear in mind. We are going to use this idea of adjusting the effect of different values to produce a combined average measure (called a *weighted mean*). We can alter the relative emphasis placed among the values by multiplying them by a set of numbers called *weights*. For example, one way of making one value twice as important as others is to multiply it by two before adding it in. The term 'weight' refers to the number attached to each item to indicate its relative importance.

Weights and weighting

Note that in this context the terms 'weight' and 'weighting' mean the same thing, but the word 'weight' rather than 'weighting' is used, because this is the term used in the calculation of the Retail Prices Index. For the remainder of this unit, then, the term 'weights' will be used in this technical sense, quite different from the everyday meaning of the weight of goods in the shopping basket as a measure of 'heaviness'.

Activity 13 Choosing a suitable weighting

Assume that the five items listed earlier make up a suitable 'basket of goods' for estimating average percentage price increases. What do you think would be a sensible set of 'weights' to choose for each item in order to calculate a meaningful average?

The most immediate weights to choose are probably the amounts of money spent on each item by a typical household over a typical week. These are the weights chosen by the government in their calculations and this is the approach adopted here.

In order to continue the discussion of the use of weights to find the 'average' price increase of a basket of goods, you need to look at the calculation of averages in general. As you will see in the next section, the idea of a weight can then be incorporated into the calculation of a particular sort of average, called the weighted mean.

Outcomes

After studying this section, you should be able to:

- ◇ appreciate the need to choose suitable 'weights' when calculating average values in certain situations (Activity 13);
- ◇ criticize the use of inappropriate measures of average price increase (Activities 11, 12).

3 A statistical interlude—averages

Aims The main aim of this section is to discuss several ways of finding averages and to introduce you to the statistical facilities of your calculator. ◇

A *batch* is the statistical term for a set of collected data.

A single number which is representative of a collection (or batch) of numbers is commonly referred to as an *average*. There are several different ways of finding such a number. Two methods are discussed briefly in Subsection 3.1: the mean and the median. Then, in Subsection 3.2, you will see how weights can be included in the calculation of an average, giving rise to the notion of a weighted mean. You will then be asked to work through two sections of the *Calculator Book*. In the first of these, you will learn how to enter the data into your calculator and use the statistical facilities of the calculator to find an 'average' using each of the two methods described earlier. The second calculator section will discuss how to include weights when using your calculator to find an average.

3.1 The mean and the median

This subsection looks at two ways of finding an 'average'. The first method produces the *mean*, which is what most people think of when they talk about an average. The second method gives the *median*, which might more accurately be described as a 'typical' or middle value. These methods will be illustrated using the following batch of heights.

The heights in metres (measured to the nearest centimetre) of a group of seven people are as follows.

1.52 1.72 1.66 1.81 1.69 1.59 1.77

The number of values in the batch (the *batch size*) is seven.

The mean

In *Unit 9*, you will come across the *geometric* mean, which is different again, so the word 'arithmetic' (pronounced with the emphasis on the 'met') is used in circumstances where the two might be confused.

As mentioned above, the *mean*, or the arithmetic mean as it is sometimes called, is what most people think of as the average of a set of numbers. It is found by adding together all the numbers in the batch and then dividing by the batch size. Thus, for the batch of heights,

$$\begin{aligned}\text{mean height} &= \frac{1.52+1.72+1.66+1.81+1.69+1.59+1.77}{7}\,\text{m}\\ &= \frac{11.76}{7}\,\text{m}\\ &= 1.68\,\text{m}.\end{aligned}$$

The median

The median is essentially the middle value of a batch (that is, the middle when the values are placed in size order); it is found in the following way.

1 First, all the values in the batch are sorted into ascending order; that is, smallest first, then second smallest, ..., with the largest last.

2 Then, see if the batch size is odd or even. If there is an odd number of values in the batch, then the middle value in the list is the median. If there is an even number of values in the batch, then there are two middle numbers. Add up these two numbers and divide by two: this gives the median. In other words, you find the mean of the two middle numbers and that gives the median of the batch as a whole.

Example 1 Finding the median when the batch size is an odd number

Find the median of the batch of seven peoples heights given previously.

1 Sorting the seven heights into ascending order produces the following list.

this is the middle value

1.52 1.59 1.66 1.69 1.72 1.77 1.81

2 There is a single middle value, so this value is the median height: 1.69 m.

Example 2 Finding the median when the batch size is an even number

Suppose that one person is removed from the batch (the tallest, for example) leaving an even number of heights (six).

1 The remaining values (in ascending order) are as follows.

these are the two middle values

1.52 1.59 1.66 1.69 1.72 1.77

2 There are *two* middle values, namely 1.66 and 1.69. The median is found by calculating the mean of these two numbers, so

$$\text{median height} = \frac{1.66 + 1.69}{2}\,\text{m} = 1.675\,\text{m}.$$

So there are two ways of finding a single number which is representative of a batch of numbers. But which should you use: the mean or the median?

And does it matter which is used? For the seven heights, the mean height (1.68 m) and the median height (1.69 m) are almost the same. So, in this case at least, it does not seem to make much difference which we use. However, to see that this is not always so, carry out the following activity.

Activity 14 *Average pocket money*

The weekly pocket money, in pounds, of each of five eleven-year-old girls is given below.

2.00 2.50 2.50 3.00 10.00

Find the mean and the median weekly pocket money of these girls. Which 'average' would you regard as the more representative of the pocket money these girls receive?

The mean weekly pocket money for the five girls is £4.00 and the median is £2.50. Clearly on this occasion, it makes much more of a difference which 'average' is chosen. On this evidence, the parents of another eleven-year-old might offer her £2.50 per week on the grounds that £2.50 (the median) is typical of what eleven-year-old girls receive in pocket money. On the other hand, the girl might argue that since the 'average' is £4.00, her parents should give her £4.00 per week!

In practice, the mean and the median are both widely used: which is chosen usually depends on the purpose for which an 'average' is required. This will be discussed further in *Unit 3*. Here in *Unit 2*, means are used throughout for finding average prices.

Now work through Section 2.1 of Chapter 2 of the Calculator Book.

Activity 15 *The mean and the median*

In the Handbook activity (Activity 9) for *Unit 1*, you wrote your own definitions of some terms. This subsection has introduced two important statistical terms—*mean* and *median*. Once again, bear in mind the advice in Activity 9 in *Unit 1* and carry out a similar task for these two important statistical terms. One aim for Block A is for you to create a glossary of important statistical terms that should prove useful as you work through the course. As well as including your own definitions of these important terms, have a go at writing down how you have 'learned' them. Try to use your own words and examples rather than just copying out definitions from the unit.

This Handbook activity will continue through the unit, so you will need to return to your printed response sheet regularly.

3.2 Calculating means using frequencies and calculating weighted means

In some situations, various values in the batch get repeated (there may be a limited number of different values that can occur, for example). It can be simpler to group the data and record the number of times (called the *frequency*) with which each different value occurs. The following example explores this possibility and comes up with an equivalent formula for calculating the mean of the batch.

Example 3 *Finding the mean household size*

Ten people were asked what size of household they lived in (that is, how many people lived in their household). They gave the following responses.

2 1 3 1 4 4 5 1 2 4

What is the mean household size?

To find the mean household size, add them all up and divide by ten.

$$\begin{aligned}\frac{2+1+3+1+4+4+5+1+2+4}{10} &= \frac{27}{10} \\ &= 2.7 \text{ people}\end{aligned}$$

However, these data can also be written in a table as follows.

The number of responses is also called the *frequency* of response.

Size of household	Number of responses
1	3
2	2
3	1
4	3
5	1

Here is the list of numbers (in ascending order) for which we have to find the mean.

1 1 1 2 2 3 4 4 4 5

As you just saw, to calculate the mean size of these households, the total number of people in all of the households (the sum of the full list of numbers) must be divided by the total number of households. There are three households of size one so, when finding the total number of people in the households, one must be counted three times (1×3); similarly, size two must be counted twice (2×2), size three once (3×1), size four three times (4×3) and size five once (5×1). Instead of writing out and adding up the full list of numbers, it can be simpler and quicker to take these five products which, when added together, give the total number of people in these households.

Recall 'product' means two or more things multiplied together.

Thus:

a household size of 4 ...

$$\text{total number of people} = (1 \times 3) + (2 \times 2) + (3 \times 1) + (4 \times 3) + (5 \times 1) = 27.$$

... occurred with a frequency of 3

Then, if you add together the numbers of responses, you get the total number of households.

$$\text{number of households} = 3 + 2 + 1 + 3 + 1 = 10$$

Dividing the total number of people in these households by the total number of households gives the mean.

$$\begin{aligned}\text{mean household size} &= \frac{(1 \times 3) + (2 \times 2) + (3 \times 1) + (4 \times 3) + (5 \times 1)}{3 + 2 + 1 + 3 + 1} \\ &= \frac{27}{10} \\ &= 2.7 \text{ people}\end{aligned}$$

Summarizing the formula for the mean household size

This method of calculating the mean may be summarized as follows.

$$\text{mean household size} = \frac{\text{the sum of the products (household size} \times \text{frequency)}}{\text{the sum of the frequencies}}$$

The *frequency* of a household size is the number of responses corresponding to that size. The sum of the frequencies is the total number of households.

One use of symbols in mathematics is in providing a more compact way of writing complicated formulas in words. One way of abbreviating this formula is to use symbols to represent 'household size' and 'frequency': for example, h for the size of a household and f for its frequency. Then the formula can be written as follows.

$$\text{mean household size} = \frac{\text{the sum of the products } (h \times f)}{\text{the sum of the frequencies } f}$$

You may find it useful to make a note in your Learning File of the meaning of the symbol $\sum$ as it is used here, so that on a future occasion you can find it easily if you have forgotten.
A copy of the Greek alphabet is included in your Learning File.

This is a little shorter, but an abbreviation for the phrase 'the sum of' would make it even shorter still. One is commonly used in mathematics: the Greek capital letter 'sigma', which is written $\sum$, and is used to mean 'the sum of'. When using symbols to represent numbers, it is not necessary to include a multiplication sign between the numbers to be multiplied: hf can be written for $h \times f$ and so $\sum hf$ means 'the sum of all the products $h \times f$'. And similarly $\sum f$ means 'the sum of all the frequencies f'. So the above formula for the mean household size may be written very concisely as follows.

$$\text{mean household size} = \frac{\sum hf}{\sum f}$$

So this symbolic formula says exactly the same as the more wordy one: add together all the products (household size × frequency) and divide by the sum of all the frequencies.

Example 4 shows how to calculate the mean household size, using sigma notation. The letter h (for household) represents household size and f (for frequency) the number of responses corresponding to each household size.

This is another example of using symbols to produce a compact record of a formula or procedure. Recall that mathematicians sometimes use the first letter of a word as a symbol to help remember what the letter stands for.

Example 4 *Using the concise formula*

Size of household (h)	Number of responses (f)	Products hf
1	3	$1 \times 3 = 3$
2	2	$2 \times 2 = 4$
3	1	$3 \times 1 = 3$
4	3	$4 \times 3 = 12$
5	1	$5 \times 1 = 5$
	$\sum f = 10$	$\sum hf = 27$

this is the sum of the frequencies

this is the sum of the hf products

To calculate the mean, divide the sum of the hf products ($\sum hf$) by the sum of the frequencies ($\sum f$).

$$\text{mean household size} = \frac{\sum hf}{\sum f} = \frac{27}{10} = 2.7 \text{ people}$$

Check through this calculation and make sure you understand the main steps and the meanings of the symbolic forms $\sum hf$ and $\sum f$.

Weighted mean

At the end of Section 2, the need for an adjusted mean—a weighted mean—arose, where different values in the batch were to be emphasized differently. The formula for the mean using frequencies can be used as a guide for obtaining the general formula for a weighted mean, because, when you have frequencies, you can put the *weights* equal to the frequency of each household size. This insight suggests that the weighted mean formula should be parallel to the frequency one, no matter what weights are used (whole numbers or decimals) or whether they are frequencies or weights derived from some other source. In general, if the weighted mean is required of a batch of numbers x with weights w, then the next formula tells you how to calculate it.

$$\text{weighted mean} = \frac{\sum xw}{\sum w}$$

That is, the weighted mean of the various x values is worked out by dividing the sum of the xw products ($\sum xw$) by the sum of weights ($\sum w$).

A general justification for using this generalized formula is beyond the scope of this course, but the following image may help you to see what is going on. The weighted mean gives the balance point for the data weighted in the corresponding way.

Suppose the data are as follows.

Value	1	2	3	4	5
Weight	3.2	2.2	1.9	3.4	1.3

The weighted mean is 2.8 (you may wish to calculate this yourself) and the image below provides an interpretation of this calculated value.

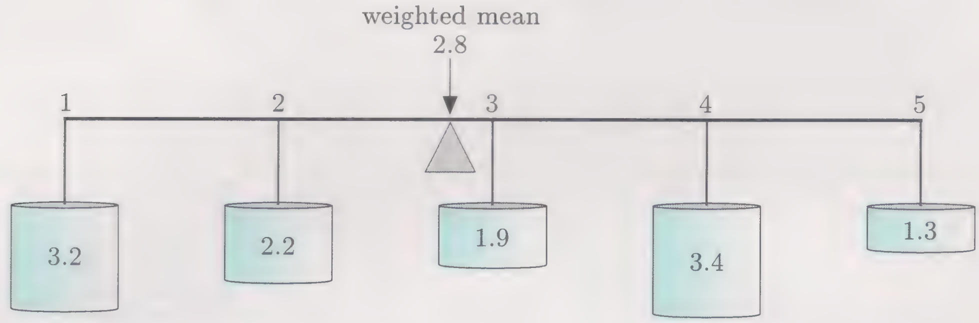


Figure 2 Picturing the weighted mean as the balance point

To return to the context of prices, Example 5 uses this formula to produce a weighted mean for the problem left unresolved at the end of Section 2.

Example 5 *The shopping basket*

Recall the shopping basket of five food items from Section 2. It was suggested that a sensible way of finding an 'average' percentage increase in the price of the shopping basket might be to weight the individual increases according to how much is spent on the items. For instance, you might choose the average amount spent on each item in a week during 1980 as its weight.

Item	Percentage increase (July 1980–July 1994)	Average 1980 weekly bill in pence (the weights)
Large loaf (white)	47	165
Milk	112	223
Eggs	89	31
Potatoes	110	46
Sugar	86	13

The amounts spent on the items will play a similar role here to that played by the frequencies in Example 4. If x is used to represent the percentage price increase of an item and w the amount spent on it by a typical household in a typical week, then a weighted mean of the percentage price increases is found by dividing the sum of the xw products ($\sum xw$) by the sum of the w values ($\sum w$).

$$\text{weighted mean percentage price increase} = \frac{\sum xw}{\sum w}$$

$$= \frac{47 \times 165 \ + \ 112 \times 223 \ + \ 89 \times 31 \ + \ 110 \times 46 \ + \ 86 \times 13}{165 + 223 + 31 + 46 + 13}$$

$$= \frac{41668}{478} = 87.17154812$$

which is an 87% increase (rounded to the nearest whole number).

What is the difference between a mean and a weighted mean?

This is a question that is commonly asked. The answer is, as usual, that it depends on the situation. The general answer is sometimes there is no difference at all, and at other times, the two formulas produce quite different values.

At its simplest, finding the mean involves adding up all the values in the batch and dividing by the number of values and can be written concisely as $\frac{\sum x}{n}$. This is the most familiar 'average'.

Grouping values that occur more than once gives rise to a different formula:

$$\frac{\sum xf}{\sum f}$$

But it will *always* give the same numerical value as the mean. This is because the sum of the frequencies ($\sum f$) is always the total number of values in the batch n.

Calculating the (ordinary) mean using frequencies provides the simplest example of a weighted mean. Only when the weights are frequencies can you be sure that the mean and the weighted mean give the same value. Otherwise they will almost certainly differ. Frequencies have two important properties: they are always whole numbers and the sum of the frequencies is always the same as the total number of values in the batch.

We used a generalized version of the frequency formula for our weighted mean formula and, as you saw in Example 5, weights need not be frequencies. There, they were the amounts of money spent in a week on different items. In this situation, there is a big difference between the mean and the weighted mean. Working out the (ordinary) mean percentage price increase for that example would mean adding 47 and 112 and 89 and 110 and 86 and then dividing by five to give 88.8%. This is quite different from calculating the weighted mean percentage price increase (of just over 87%).

Now work through Section 2.2 of Chapter 2 of the Calculator Book.

Activity 16 What is a calculator good for?

Now that you have spent a session working with your calculator, use your Learning File to record your thoughts about the calculator so far. Here are some questions you might like to consider.

- What can I use my calculator for?
- What are some benefits of using it?
- What are some drawbacks of using it?
- What am I unsure of?
- What do I need to practise?

There is a printed response sheet for this activity.

Activity 17 Weighted means

Add this to your Handbook activity sheet.

The idea of a weighted mean is rather important, so it is a good idea to pause here to be sure that you got it straight. One of the best ways of ensuring this is to try to write down for yourself what it involves, using your own words. Try doing this for the idea of a weighted mean.

You will not gain a great deal of benefit from this exercise if all you do is copy out chunks of the text. So close the unit before actually starting to write your explanation, and try to keep it closed throughout. If this proves too challenging, here is a suggestion which should help you to carry out this task for the weighted mean.

Take a few minutes to think what needs to be explained: for example, you should say what a weighted mean is; how you calculate one (perhaps with an example); what a weighted mean is for; and how it differs from other similar measures such as an ordinary mean (without frequencies). It may also make your explanation much more helpful to you personally, and more realistic, if you include some brief notes of your reactions to the ideas as you met them—for example: 'I found this bit difficult because . . .', or 'when I read this first I thought it meant . . . , but now I know a bit more I realize it means . . .'.

If you carry out this process whenever you meet an important idea in the course, you will build up your own personal glossary of mathematical concepts, which should prove extremely useful in this and later courses.

While you are assembling all this in your mind, consult the text as and when you need to. Then, when you have got it straight, close the unit and write your explanation of the term 'weighted mean'. When you are satisfied with the result, check that you have not missed anything out by referring back to the text. The words of the unit will be different from yours—after all, *your* words are best for *your* handbook entry, not ours—but just check that the facts are the same and are complete. Make any changes you think are necessary, then put your work in your Learning File.

The different definitions of the 'averages' discussed in this section are summarized in the box below. Check these with your handbook entries.

Summary of various averages

Mean	$\dfrac{\sum x}{n}$: add the x values in the batch together and divide by the batch size n.
	$\dfrac{\sum xf}{\sum f}$: for data given with frequencies, add all the products $x \times f$ and divide the result by the sum of the frequencies.
Weighted mean	$\dfrac{\sum xw}{\sum w}$: add all the products $x \times w$ and divide the result by the sum of the weights.
Median	Sort the values in the batch into ascending order (if necessary). If the batch size is odd, then the median is the middle value. If the batch size is even, then the median is the mean of the two middle values.

Outcomes

After studying this section, you should be able to:

- ◇ calculate the mean and the median of a batch of numbers;
- ◇ understand the meaning of expressions such as $\sum x$ and $\sum xw$ and use them;
- ◇ use the statistical facilities of your calculator to find the mean, the median and (given weights) the weighted mean of a batch of numbers.

Also, after working through Sections 2.1 and 2.2 of the *Calculator Book*, you should be able to:

- ◇ clear existing data lists;
- ◇ enter data;
- ◇ edit data;
- ◇ find the median and the mean of a batch of data;
- ◇ find the mean of a batch of data with frequencies.

4 Price indices

Aims The main aim of this section is to look at some different ways of measuring price increases. ◇

4.1 Back to the shopping basket

There is still the problem of calculating the average price increase of a basket of goods. Remember the problem at the end of Section 2. You found there that calculating the mean of the five price rises was unsatisfactory because the amount of each rise depended on the units in which each item was bought. For example, potatoes bought in a 1 kg bag showed a 22p increase in price. However, if the potatoes had been bought in a 50 kg sack, their price rise would become huge compared with the price rises of the other goods in the basket. Since the units in which goods are bought are fairly arbitrary, this is clearly an unsatisfactory way of calculating the average price increase of a basket of goods. To overcome this problem, we then calculated the average percentage price increase of the items.

We concluded that the most sensible set of 'weights' to choose in order to calculate a meaningful average was to weight each item by the amount of money spent on it over a typical week. We will continue to use our earlier basket of goods consisting of bread, milk, eggs, potatoes and sugar. The percentage price increases in these items between 1980 and 1994 were given in Table 8 (on page 21). So what is the weight for each of these items? That is, how much money is spent on each in a typical week?

There is a choice here. A typical week could be one in 1980 or one in 1994. Neither is wrong; either could be used: we have chosen 1980. All the relevant information is given in Table 9.

These expenditures are estimates from a particular household of their average weekly expenditure on these five items. Note that the chosen year, 1980, is clearly stated in the table.

Table 9 Percentage price increases from 1980 and expenditure

Item	% increase (1980–94)	Average 1980 weekly bill (pence)
Large loaf (white)	47	165
Milk	112	223
Eggs	89	31
Potatoes	110	46
Sugar	86	13
Total		478p

Now, calculating the *weighted mean* of the percentage price increases involves weighting each percentage figure by the corresponding amount spent on it each week. Thus, the 112% figure for milk needs to be given

the greatest weight because most money was spent on milk (£2.23), whereas the 86% figure for sugar needs to be given the smallest weight because least money was spent on sugar (13p).

Activity 18 Calculator time

Use the statistical facilities of your calculator to calculate the weighted mean of the percentage price increase from 1980 to 1994, based on these five items. The percentages are the values being considered and the 1980 average weekly expenditures are the weights.

	Values (%)	Weights
	47	165
	112	223
	89	31
	110	46
	86	13
Total of the weights		478

Note that this total, 478, is actually the £4.78 spent, on average, by this householder on these five items of food each week. In general, though, the total of the weights may not have a simple interpretation.

Check that your answer is sensible. For example, it should lie somewhere between the smallest percentage value (47%) and the largest percentage value (112%). If it lies outside this range, then you have made a mistake. So check that the total of your weights is 478.

In case you are feeling that it would have been easier to do this particular calculation directly on the calculator, there are two good reasons for working through the statistical techniques in this case. First, it is often easier to understand the principles of how such keys operate when using a simple example like this. You will need these skills in the next section when you will be doing similar calculations, but with more complex data. An appropriate household analogy might be whether it is worth getting out the sewing machine or whether you should just sew the garment by hand. The decision will probably be based on whether the job is big enough or varied enough to make it worth the effort of digging the sewing machine out from under the coats in the broom cupboard. However, if you have just bought the sewing machine, it seems sensible to learn to use it by starting on a simple task which will not create a disaster if something goes wrong; this is what you were asked to do here.

Second, even with a batch of data as tiny as this one, once the figures have been entered into the statistical memory of the calculator, a wide range of calculations and graphs is readily available. This keeps your options open if you simply want to explore different ways of summarizing and re-presenting the information.

A point worth stressing about the weights used to calculate a weighted mean is that it is not the *actual* size of the weights, but their *relative* size that determines the value of the resulting mean. For example, suppose the weights were calculated over a ten-week period and expenditure patterns remained the same. Table 10 shows what the data would look like.

Table 10 The effect of scaling up the weights by a factor of ten

Item	% increase (1980–94)	Weights
Large loaf (white)	47	1650
Milk	112	2230
Eggs	89	310
Potatoes	110	460
Sugar	86	130
Total of the weights		4780

Note that for each item the bill has simply been multiplied by ten. However, the *relative* weight of each item has not altered—the bread weight is still roughly five times the eggs weight, and so on. Thus, the value of the weighted mean will not alter. Put another way, if you had needed to feed ten times as many people with the same kind of food, you would not expect the calculation of the weighted mean of the percentage price rise of the food to come out differently. Each item has the same relative importance, regardless of the overall expenditure.

Activity 19 A weighted mean calculated over ten weeks

Calculate the weighted mean of the percentage price rise using the ten-week period expenditures as weights. Confirm that you get the same numerical answer as before when the weights were much smaller numbers. When you have done this, think about how doing this activity has contributed to your learning about weighted means.

One conclusion to draw is that it is only the relative size of the weights that determines their effect, not their absolute value.

4.2 Price ratios

Calculator Book, Section 1.4.

Here you will look at percentage price increases in a slightly different way. In Chapter 1 of the *Calculator Book*, you saw that multiplying a price by, say, 1.30 is equivalent to increasing it by 30%. Similarly, multiplying a price by 0.94 is equivalent to decreasing it by 6%. The figures 1.30 and 0.94 are called *price ratios*. The next activity should make it clear why they are called that.

Activity 20 Price ratios

Suppose that a bottle of milk rose in price from 30p to 39p.

(a) Calculate the percentage price rise.

(b) Calculate the ratio of the later price to the earlier price (that is, 39p divided by 30p).

(c) Compare the two answers and think about why the second answer is called a price ratio.

(d) How could you convert a percentage price rise into a price ratio and vice versa? (Try a few examples to get a sense of the underlying method, then try to write down a rule.)

This activity shows that information about a price change can be given either as a percentage price change or as a price ratio: the two methods are equivalent. You may care to think about whether the same is true for weighted means. Is a weighted mean of percentage price increases equivalent to the weighted mean of the corresponding price ratios? In the next activity, you are asked to do a calculation similar to the one you did in Activity 18, but this time using price ratios instead of percentage price increases, so you will be able to test whether or not the two methods produce equivalent answers. The weights involved are those used in Activity 18.

Activity 21 Finding the weighted mean of the price ratios

(a) Convert the percentage price increases in Table 9 into price ratios.

(b) Use your calculator to find the weighted mean of the price ratios for 1994 relative to 1980, based on these five items. This time the price ratios are the values being considered and the 1980 average weekly expenditures are to be used as weights.

In Activity 18, you found that the weighted mean of the percentage price increases between 1980 and 1994 was 87%. Now, in Activity 21, you found that the weighted mean of the price ratios using the same weights is 1.87. Since a price increase of 87% is equivalent to a price ratio of 1.87, this example suggests that it does not matter whether percentage price increases or price ratios are used to find the 'average' percentage price increase. It is always the case that the two answers will be equivalent and this is an important point: as you will see in Section 5, a weighted mean of price ratios is used by the British Government to assess changes in prices.

This mean price ratio can be converted directly into a percentage price increase using the method described in the comment on Activity 20.

$$\text{percentage price increase} = (\text{price ratio} - 1) \times 100\%$$

So the 'average' percentage price increase can be found either by using percentages or by using price ratios. This illustrates a general point about mathematics, namely that there is often more than one way to perform a calculation.

4.3 Price indices

Cast your mind back to why percentages were introduced in Section 2. It was because using actual price changes is unsatisfactory in comparing how the prices of two different items have altered over time when their basic prices are very different. For example, if the price of a motor car has gone up by £100 and the price of a bicycle has gone up by £50 over the same period, which is the more significant price rise? Expressed in terms of its basic price, the bicycle price increase is much the greater of the two and any sensible comparison of the two price increases must take that into account. In other words, comparisons must be based on relative, not absolute, differences.

Two measures based on relative comparisons have been introduced already, namely percentage price increases and price ratios. The tape session which follows introduces a third measure called a *price index*.

In the next section, you will read about the Retail Prices Index; this is a price index used to measure overall price changes in the UK.

Index

The word 'index' has a number of different but connected meanings. An *index* appears at the back of a book to help you find what is in it. The *Index* was a list of prohibited books that Roman Catholics were not to read. The *index* finger (next to the thumb) is for pointing things out. The plural of the word 'index', used mathematically, is 'indices' (just as the plural of vertex, a corner of a geometric figure made of straight lines, is vertices). This makes it easier to see the link with words such as 'indicative' and 'indicator': the indicator on a car informs others about where you are going.

The speaker on the audiotape you are about to listen to talks about building a price index and about price indicators: both technical usages of these terms spring from the same root word. In mathematics, you may also have come across the word 'index' used as another name for the exponent (or power), the number to which another number is raised.

Listen to band 3 of Audiotape 1. You will need your calculator, pen and paper as you work through the tape.

Frame 1

January bread prices

Average January bread prices for a large white sliced loaf

Year	1987	1988	1989	1990	1991	1992	1993	1994
Price (p)	44	46	48	50	53	54	55	51

Frame 2

Calculating price ratios relative to January 1987

The price ratio for January 1988 relative to January 1987 is

$$\frac{\text{average price in January 1988}}{\text{average price in January 1987}} = \frac{46}{44} \simeq 1.045$$

These have been rounded to three decimal places

The price ratio for January 1989 relative to January 1987 is

$$\frac{\text{average price in January 1989}}{\text{average price in January 1987}} = \frac{48}{44} \simeq 1.091$$

Frame 3

Price ratios relative to January 1987

Year	1987	1988	1989	1990	1991	1992	1993	1994
Price (p)	44	46	48	50	53	54	55	51
Price ratio	1.000	1.045	1.091					

Frame 3A

Price ratios relative to January 1987

Year	1987	1988	1989	1990	1991	1992	1993	1994
Price ratio	1.000	1.045	1.091	1.136	1.205	1.227	1.250	1.159

Frame 4

A price index for bread: the base year method

The base year …

Year	1987	1988	1989	1990	1991	1992	1993	1994
Index	100	104.5	109.1	113.6	120.5	122.7	125.0	115.9

… has a value of 100

100 × price ratio relative to base year

Frame 5

Using a price index to calculate price changes

Calculation A

Using the bread prices

$$\text{Price ratio for 1990 relative to 1988} = \frac{\text{average price in January 1990}}{\text{average price in January 1988}} = \frac{50}{46} \simeq 1.087$$

Calculation B

Using the price index

$$\text{Price ratio for 1990 relative to 1988} = \frac{\text{value of price index in 1990}}{\text{value of price index in 1988}} = \frac{113.6}{104.5} \simeq 1.087$$

$$\boxed{\text{percentage price increase} = (\text{price ratio} - 1) \times 100\,\%}$$

Between January 1988 and January 1990 the average price of a large white sliced loaf rose by $(1.087 - 1) \times 100\,\% = 8.7\%$

Frame 6

Exploring price indices—an exercise

Use the price index in Frame 4 to calculate the percentage increase in the average price of a large white sliced loaf

(a) between January 1987 and January 1988

(b) between January 1987 and January 1990

(c) between January 1990 and January 1993

There are comments after Comments on Activity 21

Frame 7

One-year price ratios

Prices are in Frame 1

$$\text{Price ratio for 1989 relative to 1988} = \frac{\text{average price in January 1989}}{\text{average price in January 1988}} = \frac{48}{46} \simeq 1.043$$

Year	1987	1988	1989	1990	1991	1992	1993	1994
One-year price ratio	1.000	1.045	1.043					

Frame 7A

One-year price ratios for a large white sliced loaf

Year	1987	1988	1989	1990	1991	1992	1993	1994
One-year price ratio	1.000	1.045	1.043	1.042	1.060	1.019	1.019	0.927

Frame 8

A price index for bread: the previous year method

Examples:

$$\text{Price ratio for 1989 relative to 1988} = \frac{\text{value of price index in January 1989}}{\text{value of price index in January 1988}}$$

$$\text{Price ratio for 1990 relative to 1989} = \frac{\text{value of price index in January 1990}}{\text{value of price index in January 1989}}$$

In general:

$$\text{Price ratio for given year relative to previous year} = \frac{\text{value of price index in given year}}{\text{value of price index in previous year}}$$

$$\boxed{\text{value of price index in given year} = \text{value of price index in previous year} \times \text{price ratio for given year relative to previous year}}$$

$$\begin{aligned}\text{value of price index in 1988} &= \text{value of price index in 1987} \times \text{price ratio for 1988 relative to 1987}\\ &= 100 \times 1.045\\ &= 104.5\end{aligned}$$

$$\begin{aligned}\text{value of price index in 1989} &= \text{value of price index in 1988} \times \text{price ratio for 1989 relative to 1988}\\ &= 104.5 \times 1.043\\ &\simeq 109.0\end{aligned}$$

Values of price index for bread

Year	1987	1988	1989	1990	1991	1992	1993	1994
Index	100	104.5	109.0					

Frame 8A

Values of price index for bread

The base year ...

Year	1987	1988	1989	1990	1991	1992	1993	1994
Index	100	104.5	109.0	113.6	120.4	122.7	125.0	115.9

... is given a value of 100

Frame 9

Rounding errors

Values of price index

Year	1987	1988	1989	1990	1991	1992	1993	1994
Base year method	100	104.5	109.1	113.6	120.5	122.7	125.0	115.9
Previous year method	100	104.5	109.0	113.6	120.4	122.7	125.0	115.9

discrepancies due to rounding error

Example

Find 1.22×3.12 rounded to one decimal place.

1 Multiplying unrounded numbers:

$$1.22 \times 3.12 = 3.8064 \simeq 3.8$$

discrepancy due to rounding error

2 Multiplying rounded numbers:

$$1.2 \times 3.1 = 3.72 \simeq 3.7$$

A price index for the shopping basket

On the audiotape, two methods of constructing a price index for bread were described. They were called the 'previous year' method and the 'base year' method. Either method could be used to construct a price index for the shopping basket of five items in Table 9. We use the 'previous year' method here. The value of the index in the base year is 100. Then each year the value of the index for the previous year is multiplied by the basket price ratio for the year to obtain the new value of the index.

The basket price ratio is the weighted mean of the price ratios for the five items: the weights used are proportional to the amounts spent on each

item in a typical week. Recall from Subsection 4.1 that it is only the relative size of the weights that determines their effect, not their absolute value. So as long as the relative size of the amounts spent on the items remains unchanged from year to year, there is no need to change the weights. For simplicity, assume that the various weights remained unchanged between 1987 and 1994. The weights used are for the two-person household of a member of the course team. Table 11 shows the price ratios for the five items for January 1988 relative to January 1987 and the weights used.

Table 11 Price ratios and weights for the shopping basket

Item	Price ratio	Weight (average 1987 weekly bill in pence)
Large loaf	1.045	180
Milk	1.040	300
Eggs	1.086	35
Potatoes	1.000	50
Sugar	1.106	15

Activity 22 *The basket price ratio*

Find the price ratio for the basket for January 1988 relative to January 1987 by finding the weighted mean of the five price ratios in Table 11.

Calculations similar to the one in this activity resulted in the year-on-year basket price ratios shown in Table 12.

Table 12 Year-on-year price ratios for the shopping basket

Year	1987	1988	1989	1990	1991	1992	1993	1994
Price ratio	1.000	1.043	1.050	1.065	1.070	1.038	1.005	1.016

Taking 1987 as the base year, the value of the price index (generally simply called the index) is set to 100 in January 1987. The value of the index in January 1988 is then

$$\begin{aligned}&100 \times \text{price ratio for January 1988 relative to January 1987}\\&\quad = 100 \times 1.043\\&\quad = 104.3.\end{aligned}$$

As was the case for the index for bread, each year the value of the index is found by multiplying the value of the index a year earlier by the price ratio for the year. So the value of the index for January 1989 is given by the following computation.

$$\begin{aligned}\begin{pmatrix}\text{value of index in}\\\text{January 1988}\end{pmatrix} \times \begin{pmatrix}\text{price ratio for January 1989}\\\text{relative to January 1988}\end{pmatrix} &= 104.3 \times 1.050\\&\simeq 109.5.\end{aligned}$$

Activity 23 Calculating the price index

Calculate the value of the price index for the shopping basket in Table 11 for each of the years 1990 to 1994 using the price ratios in Table 12.

Remember that you saw how the value of the index for 1989 was calculated in the last paragraph.

The price index is simply the price ratio for the period since the base year, multiplied by 100. The value of the price index was 116.6 in January 1990, so the basket cost 1.166 times as much in January 1990 as in January 1987. That is, the price of the basket had risen by 16.6%. The percentage increase in the price of the basket between any two years can be found by finding the ratio of the value of the index in the later year to the value in the earlier year, and then converting this price ratio to a percentage price rise. For example, the price ratio for January 1991 relative to January 1988 is worked out as follows.

$$\frac{\text{value of index in January 1991}}{\text{value of index in January 1988}} = \frac{124.8}{104.3} \simeq 1.197$$

So the price of the basket rose by 19.7% between January 1988 and January 1991.

Activity 24 Calculate percentage price changes

(a) Find the percentage rise in the price of the shopping basket between January 1990 and January 1994.

(b) Find the percentage rise in the price of the shopping basket between January 1989 and January 1992.

The basket price ratio was found by weighting the price ratios of the five items according to how much was spent on them in a typical week. For simplicity, when calculating the index, the relative amounts spent each week on the items in the basket were assumed to remain unchanged over the years. However, this is not always a reasonable assumption. For example, over a number of years, less may be spent on milk and more on eggs. This information could be incorporated into the calculation of the index by changing the weights used to calculate the basket price ratio each year to reflect changes in spending. As you will see, this is what happens for the Retail Prices Index. This is an advantage of building a price index by calculating one-year price ratios (the previous year method) over calculating price ratios relative to the base year (the base year method): the former allows changes in spending patterns to be taken into account.

Of course, this basket of five items is not really very representative of all the expenditure made in a typical week. As you will see in the next section, many more goods and services are taken into account by the government in the calculation of the Retail Prices Index.

This would be a good time to check that you have a record of each activity that you have completed in this section. And are there any terms or ideas that you feel it would be useful for you to make notes on to include in your handbook? When you have made any notes you wish to, check your handbook entries with the information in the box below. The main points about the construction of a price index whose value is calculated once a year are summarized there.

'Price index' might be a good one. Add this to your Handbook activity sheet.

A price index

The starting value of the index (that is, in the base year) is 100.
Each year, the new value of the index is calculated using the following formula.

$$\left(\begin{array}{c}\text{value of price index}\\ \text{in one year}\end{array}\right) = \left(\begin{array}{c}\text{value of price index}\\ \text{in previous year}\end{array}\right) \times \left(\begin{array}{c}\text{price ratio for the}\\ \text{year relative to}\\ \text{the previous year}\end{array}\right)$$

Using a price index

The percentage price increase between any two years covered by an index can be found by first finding the price ratio for the later year relative to the earlier year; this is given by the following formula.

$$\text{price ratio} = \frac{\text{value of price index in later year}}{\text{value of price index in earlier year}}$$

Then the percentage price increase is given by this final formula.

$$\text{percentage price increase} = (\text{price ratio} - 1) \times 100\%$$

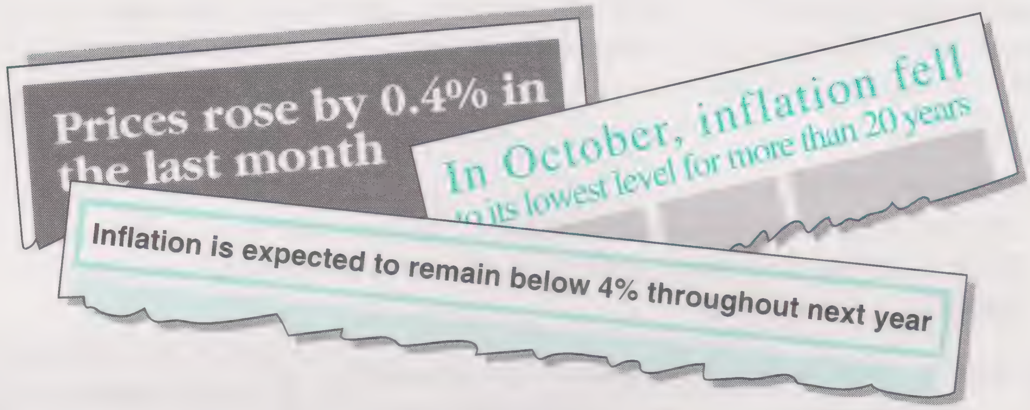
Outcomes

After studying this section, you should be able to:

- ◇ use a weighted mean to find an average percentage price increase or price ratio (Activities 18, 19, 21);
- ◇ convert a percentage price increase into a price ratio or index and vice versa (Activities 20, 22);
- ◇ calculate a percentage price increase and a price ratio from a price index (Frame 6, Activity 24);
- ◇ use price ratios to work out values of a price index (Frame 8, Activity 23).

5 The Retail Prices Index

Aims The main aim of this section is to discuss what the Retail Prices Index measures and how it is calculated. ◇



How often have you heard statements like these on the radio or read them in the newspapers? Have you ever wondered how 'inflation' is measured or precisely what is meant by a statement such as 'prices rose by 0.4%'? In Section 6, you will see that 'rates of inflation' are calculated in the UK using an index of retail prices, the Retail Prices Index (RPI). This index may also be used to calculate the percentage by which prices in general have risen over any given period. But what exactly does this index measure and how is it calculated? These are the questions that are addressed in this section.

Activity 25 RPI

Before reading about the Retail Prices Index and how it is used to measure price changes, pause for a moment to think about what you already know about the RPI. Write down what you think the RPI is and how you think it might be used to measure 'inflation'. (Use your Handbook activity sheet.) Later in the unit, you will be asked to look back at what you write now and to add to your notes in the light of what you have learned by working through this section and the next.

5.1 What is the RPI?

The Retail Prices Index (RPI), which is published each month by the UK Central Statistical Office, is the main measure used in the UK to record changes in the level of the prices most people pay for the goods and services they buy. The RPI is intended to reflect the average spending pattern of the great majority of households. Only two classes of households are excluded, on the grounds that their spending patterns differ

greatly from those of the others: pensioner households and high-income households. Separate retail price indices are calculated quarterly for one-person and for two-person pensioner households. In order to distinguish the RPI from these other indices, the main index is often referred to as the *general index* of retail prices.

Roughly the top 4% are deemed 'high income' households.

The households covered by the general index are known as *index households.* All three indices are published in the *Employment Gazette*, a Department for Education and Employment monthly publication. This is available in many public libraries. If you go to a library, try to have a look at a copy. The figures for the general index (referred to simply as the RPI) are currently given in Table 6.4 of each edition: find the most recent value of the RPI in the current edition and make a note of it. When you have completed this unit, you may be able to use this value to work out for yourself how much prices have changed since this unit was written in 1994.

Since this course was written, the *Employment Gazette* has been discontinued. All information is now included in the monthly publication *Labour Market Trends.* Table numbers for the RPI (and the AEI, referred to in *Unit 3*) are unchanged.

You will not be able to do this if there have been any changes to the way the RPI is calculated since 1994.

The RPI is calculated in essentially the same way as the price index for the shopping basket of five items described in Subsection 4.3. However, it is calculated once a month instead of once a year, and it is based on a very large 'basket' of goods. The contents of the basket and the weights assigned to the items in the basket are updated once a year to reflect changes in spending patterns. However, once decided on at the beginning of the year, they remain fixed throughout the year. Each month the price ratio for the basket is calculated relative to the previous January. Then the value of the index is obtained by multiplying the value of the index for the previous January by this price ratio. For example,

$$\text{RPI for May 1994} = \text{RPI for January 1994} \times \begin{pmatrix} \text{price ratio for May 1994} \\ \text{relative to January 1994} \end{pmatrix}$$

Since the RPI is calculated from price ratios, it measures price changes in terms of the *ratio* of the overall level of prices in a given month to the overall level of prices at an earlier date. In practice, data on prices are collected on a day near the middle of the month; the value of the RPI calculated using these data is referred to simply as the value of the RPI for the month. The value of the index for a particular month—for example, 141.3 in January 1994—measures the ratio of the overall level of prices in that month to the overall level of prices on a date at which the index was fixed at its starting value of 100. This date is called a *base date.*

In 1994, the base date for the RPI was 15 January 1987. Thus the general level of prices in January 1994, as measured by the RPI, was $\frac{141.3}{100} = 1.413$ times the general level of prices in January 1987; or, equivalently, prices in January 1994 were 41.3% higher than in January 1987. The base date has *no* significance other than to act as a reference point.

The RPI is based on a very large 'basket' of goods. This basket does not include every item bought by index households: this would not be practicable. A selection has to be made, but even so, the basket is very large. Some six hundred items are chosen, including most of the usual things people buy—food, clothes, fuel, household goods, housing, transport, services, and so on. The basket is an 'average' basket for a broad range of households.

The items in the basket are often grouped into broader categories. The five fundamental groups are 'Food and catering', 'Alcohol and tobacco', 'Housing and household expenditure', 'Personal expenditure' and 'Travel and leisure'. These can be divided into fourteen more detailed subgroups, which in turn may be split into ninety-five sections. Finally, the sections are comprised of the six hundred or so individual items on which the RPI is actually calculated. This is illustrated in Figure 3.

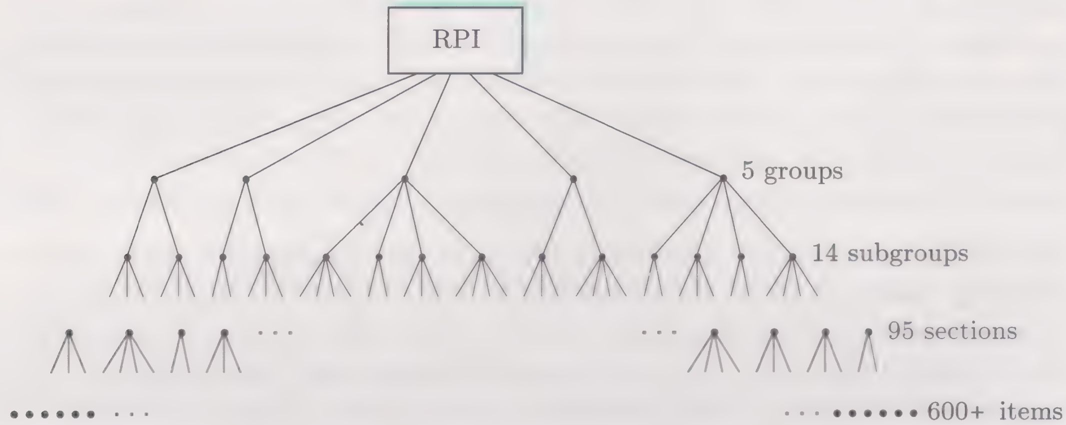


Figure 3 Levels of grouping of the RPI

Figure 4 opposite shows most of the ninety-five sections; it gives the names of the five groups, the fourteen subgroups and the most important sections. The inner circle shows the five groups, the outer ring shows the fourteen subgroups, and the boxes contain the names of the most important sections. The structure remains the same at the time of writing (1994), although there have been some small changes in the weights since 1987. For example, the weight of 'Food and catering' is smaller now than in 1987, and the weight of 'Travel and leisure' is larger. Notice that in the inner circle the sector labelled 'Food and catering' has been drawn almost twice as large (as measured by area) as that labelled 'Alcohol and tobacco'. This reflects the fact that the typical household spends nearly twice as much on food and catering as on alcohol and tobacco. The weight of an item or group reflects how much money is spent on it. So the weight of the 'Food and catering' group is almost twice that of 'Alcohol and tobacco'.

The outer ring represents the same total expenditure as the inner circle, but in more detail. For example, in the outer ring the area labelled 'Food' is more than three times as large as that labelled 'Catering', reflecting the fact that the typical household spends more than three times as much on food as on catering; the weight of the subgroup 'Food' is more than treble the weight of the subgroup 'Catering'. The chart gives a good indication of average spending patterns in the UK in the late eighties and early nineties. Study the chart for a minute or two to get a sense of the relative weightings of the groups and of the subgroups.

Structure of the Retail Prices Index in 1987

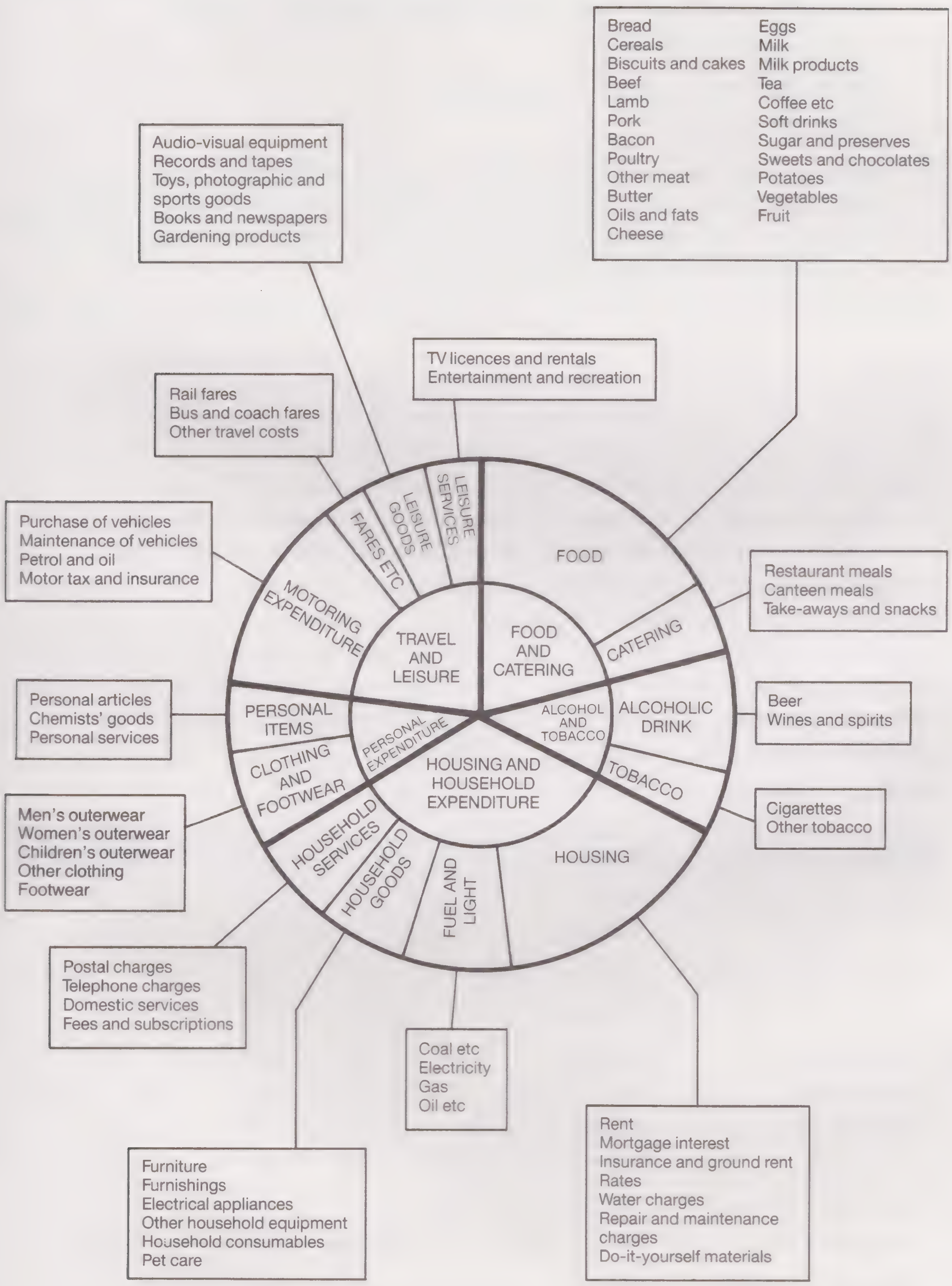


Figure 4

Source: A short guide to the Retail Prices Index, 1987, HMSO

Activity 26 *Exploring the RPI shopping basket*

(a) Using Figure 4, estimate what fraction of the expenditure of a typical household is on each of the following groups and subgroups:

(i) personal expenditure;

(ii) housing and household expenditure;

(iii) housing.

(b) Suppose that a household spends a total of £240 per week on goods and services that are covered by the RPI. Use your answers to part (a) to estimate very approximately how much is spent each week on each of the groups and subgroups in part (a).

To ensure that the index basket reflects the proportion of average spending devoted to different types of goods and services, it is necessary to find out how people actually spend their money. The Family Expenditure Survey (FES), which has been carried out continuously since 1957, records the spending reported by a sample of some seven thousand households spread throughout the United Kingdom. It provides much information on household spending and income, and has many uses. The most important use remains the provision of information on spending patterns used in compiling the RPI. Data from the FES are used to calculate the weights of the items included in the basket. Since 1962, the weights have been revised each year, so that the index is always based on a basket of goods and services which is as up to date as possible.

Table 13 1994 weights

Group	subgroup	Weight	Group weight
Food and catering	Food	142	
	Catering	45	187
Alcohol and tobacco	Alcoholic drink	76	
	Tobacco	35	111
Housing and household expenditure	Housing	158	
	Fuel and light	45	
	Household goods	76	
	Household services	47	326
Personal expenditure	Clothing and footwear	58	
	Personal goods and services	37	95
Travel and leisure	Motoring expenditure	142	
	Fares and other travel costs	20	
	Leisure goods	48	
	Leisure services	71	281
All items (i.e. the sum of the weights)			1000

The weight of an item directly depends on the average expenditure of index households on that item. In the first subsection of Section 4, you saw that it is only the *relative* size of the weights that affects the value of the weighted mean: multiplying all the weights by ten, for instance, did not affect the value of the weighted mean. So instead of using the average expenditure of an item as its weight, the expenditure figures for the items can all be multiplied by the same factor to produce a new set of weights. For the RPI, this factor is chosen for simplicity so that the sum of the weights is 1000. (Remember it is only the *relative* size of the weights that matters.) Table 13 shows the 1994 weights for the groups and subgroups. Notice that each group weight is obtained by adding the weights for all the subgroups in that group. Similarly, the weight of each subgroup is the sum of the weights of the sections in that subgroup.

In order to develop your understanding of what the weights represent, and to enable you to judge how well the group weights used in the RPI represent your particular household's spending patterns, spend a few minutes thinking about how much you spend each *month* on average on a number of different items. *Do not take more than fifteen minutes on this*;it could

take you quite a long time to produce accurate figures, which are not needed here. The list provided in Table 14 (overleaf) contains the major categories of goods included in the RPI, so you should be able to make reasonable estimates of your household's group weights by completing the last three columns of the table. Some items may not form a regular part of your *monthly* expenditure: if that is the case, then just make a rough estimate of, say, your annual expenditure and divide by twelve.

Table 14 was completed for a two-person household. Some of the figures were accurate, others were necessarily very rough estimates. The £947 total represents the average monthly expenditure for this particular household. Nevertheless, the household's weights give a good indication of the proportion of the household's expenditure (in 1993) on the five main groups used in the RPI. The weight for 'Food and catering', for example, was obtained by multiplying the proportion of monthly expenditure spent on food and catering by 1000. The proportion of monthly expenditure on 'Food and catering' is

$$\frac{285}{947} \simeq 0.301.$$

Since the total weight is 1000, the weight for 'Food and catering' is

$$0.301 \times 1000 = 301.$$

Notice that the group weights for this particular household differ quite considerably from those used in the RPI in 1994. In particular, a much greater proportion of expenditure is on 'Food and catering' and a much smaller proportion is spent on 'Alcohol and tobacco'.

Table 14 A checklist for one household's average monthly expenditure

	Expenditure and weights			Your expenditure and weights		
	Expenditure 1993 (£)	Group totals (£)	Group weights	Expenditure last year (£)	Group totals (£)	Group weights
Food and catering						
—at home	230			—		
—canteens, snacks and take-aways	45			—		
—restaurant meals	10			—		
		285	301		___	___
Alcohol and tobacco						
—alcoholic drink	5			—		
—cigarettes	0			—		
		5	5		___	___
Housing and household expenditure						
—mortgage interest/rent	65			—		
—council tax	41			—		
—water rates	23			—		
—house insurance	26			—		
—repairs/maintenance/DIY	20			—		
—gas/electricity/coal/oil bills	90			—		
—household goods (furniture, appliances, consumables, etc.)	40			—		
—telephone bills	13			—		
—pet care	0			—		
		318	336		___	___
Personal expenditure						
—clothing and footwear	50			—		
—other (hairdressing, chemists' goods etc.)	6			—		
		56	59		___	___
Travel and leisure						
—motoring (purchase, maintenance, petrol, tax, insurance)	130			—		
—fares	100			—		
—books, newspapers, magazines	40			—		
—toys, photographic and sports goods	3			—		
—TV purchase/rental, licence	0			—		
—cinema, theatre, etc.	10			—		
		283	299		___	___
		947	1000		___	___

Activity 27 Finding your household's group weights

Complete the final three columns of the checklist in Table 14 and thus estimate your household's monthly expenditure last year on each of the five main groups and your household's group weights. *If you really do not know how much you spend on a category, then make a sensible guess—exact figures are not important.* If you have no idea at all, then use the corresponding figure in Table 14 as a starting point for your own and adjust it up or down depending on how you think you spend your money. One way of checking that your figures are sensible is to make sure that the sum of the expenditures does not exceed your household's monthly income!

How do your household's weights compare with those used in the RPI in 1994? In particular, assuming your household is not an index household, in what way do your household's weights differ from the 1994 RPI weights?

Activity 28 The basket of goods

The purpose of this activity is to help you to describe in your own words some of the important points about the RPI that have been discussed so far in this section. You should write your answers in several sentences, to form a clear summary of these points. Do not look at the comments on this activity until you have also done Activity 29.

The RPI reflects changes in the price of a fixed basket of goods.

(a) What does this basket of goods represent?

(b) What do the weights of the items in the basket represent?

(c) Why is it necessary to change the contents of the basket from time to time?

(d) What information is used to obtain the weights of the items in the basket?

Activity 29 Being self-critical about your writing

Pause now and focus your attention on how you set about tackling the last activity. *Before* looking at the comments and reading on, *take a minute or two to read what you have written.* Then ask yourself the following questions.

◇ Is what you have written clear?

◇ Are your answers informative? (That is, are they self-explanatory? Or would you have to read the question in order to understand what they are trying to say?)

- ◇ Does what you have written actually make sense?
- ◇ And finally, have you actually answered the question asked or a slightly different one?

As you meet new ideas and develop problem-solving skills, you should also at the same time develop the skill of communicating your ideas clearly and concisely. Solving a problem is not a very useful activity in itself unless you can explain what you have done and report your results to others. The main way to develop these communication skills is to practise them. As you work through the course, you will be given many opportunities to do so. Responding to the questions given above will give you a systematic way of looking at your answers critically and hence help you to improve your communication skills. You should also find that writing a clear explanation of a problem and its solution or a brief summary of facts will help you to learn more effectively.

Finally, check your responses to Activity 28 against the above set of questions, and amend your answers where necessary. Now take a look at the comments to that activity on page 81.

5.2 Calculating the index

To measure price changes in general, it is not necessary to obtain prices for all the possible items of goods and services available to people. It is sufficient to select a limited number of representative items to indicate the price movements of a broad range of similar items. As previously mentioned, the index basket is chosen to be representative of all items bought by index households. For each of the ninety-five sections of the RPI, a number of items are selected for pricing (called the *price indicators*). The selection is made in such a way that the price movements of the items selected, when combined using a weighted mean, provide a good estimate of price movements in the section as a whole. Thus, although not all items of goods and services are priced, the index still measures price changes for the whole range of consumer goods and services.

For example, in 1994 the price indicators in the 'Bread' section (which is contained in the 'Food and catering' group) were the five types of loaf shown in Table 3: large white sliced, large white unwrapped, small white unsliced, small brown sliced and large brown unsliced. Changes in the prices of these loaves (when combined using a weighted mean) are assumed to be representative of changes in bread prices as a whole. Note that although the *price ratio* for bread is based on this small sample of five types of bread, the calculation of the appropriate *weight* for bread is based on *all* kinds of bread, only five of which are price indicators. This weight is calculated using data collected in the Family Expenditure Survey.

The Family Expenditure Survey is discussed more in *Unit 3*.

As the index is intended to measure price changes, it is important to collect price information for exactly the same goods and services every month—that is, for the same quantity, for the same brand, in the same place. It would, for instance, be wrong to include a price change obtained from comparing a cut-price brand of flour with a more expensive one, or from using a corner shop one month and a supermarket the next. The items, whose prices are recorded, are selected at the beginning of the year and remain exactly the same throughout the year.

Collecting the data

The bulk of the data on price changes required to calculate the RPI is collected by staff of the Department for Education and Employment, and forwarded to the Central Statistical Office for processing. Collecting the prices is a major operation: some 150 000 prices are collected each month. The prices being charged at a large range of shops and other outlets throughout the UK are all recorded on the same day (a predetermined Tuesday near the middle of the month).

Once the data have been collected, the process of calculating the RPI begins. The main steps involved in the calculation are discussed on the audiotape sequence that you should listen to next. You will need to use your calculator to find weighted means. If you are not sure how to do this, it would be a good idea to look again at Section 2.2 of Chapter 2 of the *Calculator Book* before you begin the audiotape session.

Now listen to band 4 of Audiotape 1. You will need your calculator, pen and paper as you work through the audiotape.

AUDIO

Frame 1

The data

The RPI is calculated once a month using weights and prices.

Weights

- based on FES data
- updated once a year

Prices

- for 600+ items—the price indicators
- collected once a month by Department of Employment staff

Frame 2

Once the data have been collected ...

1 Credibility tests—prices are vetted to identify gross errors.

e.g. 5p for a loaf which generally costs 55p
£3.00 for a pint of milk which generally costs 30p

2 An average price (a weighted mean) is calculated for each item.

3 The price ratio is calculated for each item relative to the previous January.

e.g. for April 1994, the price ratio for an item relative to the previous January is

$$\frac{\text{average price in April 1994}}{\text{average price in January 1994}}$$

4 Weighted means are used to calculate in turn:

- section price ratios
- subgroup price ratios
- group price ratios
- the all-item price ratio

The basket price ratio

All price ratios are relative to the previous January

For April 1994, the 1994 weights are used.

5 $$\text{Value of RPI for April 1994} = \text{value of RPI for January 1994} \times \text{all-item price ratio}$$

Frame 3

Calculating the value of the RPI for April 1994

Value of RPI in January 1994 = 141.3

Group	Price ratio r	Weight w	Ratio × weight rw
Food and catering	1.014	187	189.618
Alcohol and tobacco	1.006	111	111.666
Housing and household expenditure	1.032	326	336.432
Personal expenditure	1.032	95	98.040
Travel and leisure	1.012	281	284.372
Sum		1000	1020.128

This is the Greek capital letter sigma which means 'the sum of'

$$\text{All-item price ratio} = \frac{\Sigma rw}{\Sigma w} = \frac{1020.128}{1000}$$

$$= 1.020128$$

$$\text{Value of RPI for April 1994} = \text{value of RPI for January 1994} \times \text{all-item price ratio}$$

$$= 141.3 \times 1.020128$$

$$= 144.1440864$$

$$\simeq 144.1$$

Frame 4

Price ratios and weights

From February 1994 to January 1995, calculation of the RPI uses:

- price ratios relative to January 1994
- 1994 weights

From February 1995 to January 1996, calculation of the RPI uses:

- price ratios relative to January 1995
- 1995 weights

Frame 5

An exercise

Find the value of the RPI in March 1994 and in June 1994 given the information in the table below. The value of the RPI in January 1994 was 141.3.

There are comments after Comments on Activity 29

Group	Price ratio relative to January 1994 March 1994	June 1994	1994 weights
Food and catering	1.011	1.022	187
Alcohol and tobacco	1.003	1.011	111
Housing and household expenditure	1.004	1.034	326
Personal expenditure	1.029	1.034	95
Travel and leisure	1.008	1.015	281

All-item price ratio for March 1994 relative to January 1994 =

value of RPI in March 1994 = ×

=

≃

All-item price ratio for June 1994 relative to January 1994 =

value of RPI in June 1994 = ×

=

≃

The calculation of the RPI in Frames 3 and 5 of the tape session was carried out using the five groups. However, when answering questions about the effect of individual changes on the overall level of the RPI, you should remember where the price ratios for these groups come from. Each group price ratio is actually a weighted mean of price ratios for the subgroups which make up the group; and each subgroup price ratio is a weighted mean of section price ratios. The price ratios for the individual items in a section are combined using a weighted mean to give the section price ratio. So the change in price of any single item is likely to have only a small effect on the all-item price ratio and hence on the RPI.

The following activity is intended to help you to draw together many of the ideas you have met in this section, both about what the RPI is and how it is calculated. Take your time over it; it will require some careful thought. You may find that you need to refresh your memory about some of the details about weights and price changes: take the time to do this.

Activity 30 *Estimating the impact of different price changes*

Between July 1993 and July 1994, the price of food rose on average by 0.8%, while the price of furniture fell by 1%. Answer the following questions about the likely effects of these changes on the value of the RPI. (No calculations are required.)

(a) Looked at in isolation (that is, supposing that no other prices changed), would the change in the price of food lead to an increase or a decrease in the value of the RPI?

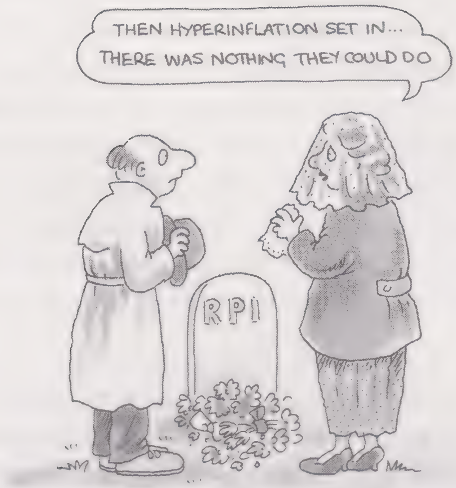
Would the change in the price of furniture (looked at in isolation) lead to an increase or a decrease in the value of the RPI?

(b) In each case, is the size of the increase or decrease likely to be large or small?

(c) Using what you know about the structure of the RPI, decide which of 'Food' and 'Furniture' has the larger weight.

(d) Which of the price changes mentioned in the question will have a larger effect on the value of the RPI? Briefly explain your answer.

(e) Can you list some different strategies you have used to learn about the RPI?



Outcomes

After studying this section, you should be able to:

- ◇ identify and summarize the key points in a piece of text in your own words and constructively criticise your account (Activity 28, 29);
- ◇ calculate the value of the RPI given the relevant information; for example, price ratios (Frame 5);
- ◇ describe the likely effect of particular price changes on the value of the RPI (Activity 30);
- ◇ explain the principles behind the calculation of the RPI to someone else (Activity 28).

If you wish to find out more about the RPI, it is discussed in greater detail in Block A of the Open University course MDST242 *Statistics in Society*.

Before reading on, check that you have a record of each activity that you completed in this section and put them in your Learning File.

6 Using the RPI

Aims The aim of this section is to discuss various uses of the RPI. ◇

6.1 Three uses of the RPI

You will need your calculator, pen and paper as you work through the section.

The RPI is intended to help measure price changes. How it is used to do this is discussed in the audiotape band which follows.

Now listen to band 5 of Audiotape 1.

Frame 1

The news headline

Radio 4: World at One: 16 February 1994

'Inflation was running at an annual rate of 2½% in January, up from 1.9% in December'

Frame 2

The annual rate of inflation

Annual rate of inflation = percentage increase in value of RPI over a year earlier

Frame 3

The data

Date	December 1992	December 1993	January 1993	January 1994
RPI	139.2	141.9	137.9	141.3

Frame 4

The headline rate explained

$$\text{Price ratio} = \frac{\text{value of RPI in December 1993}}{\text{value of RPI in December 1992}} = \frac{141.9}{139.2} \simeq 1.019$$

RPI in December 1993 = 101.9% of RPI in December 1992
RPI increased by 1.9% between December 1992 and December 1993
Annual rate of inflation in December 1993 was 1.9%

year-on-year rate of inflation = annual rate of inflation = headline rate of inflation

Frame 5

Over to you

$$\text{Price ratio} = \frac{\text{value of RPI in January 1994}}{\text{value of RPI in January 1993}} = \frac{\quad}{\quad} \simeq \quad$$

RPI in January 1994 = ______ of RPI in January 1993

Annual rate of inflation in January 1994 was ______

Frame 6

Exercise 1

The value of the RPI in July 1994 was 144.0; its value in July 1993 was 140.7. Find the annual rate of inflation in July 1994.

Frame 7

Index-linked pensions

Q: Why index-link a pension?

A: So that, as prices rise, it continues to pay for the same quantity of goods and services.

Q: How is index-linking done?

A: By increasing the pension by the same percentage as the percentage rise in prices, or equivalently, use these formulas

$$\text{pension at later date} = \text{pension at earlier date} \times \text{price ratio}$$

$$\text{pension at later date} = \text{pension at earlier date} \times \frac{\text{RPI at later date}}{\text{RPI at earlier date}}$$

Frame 8

Examples

1 A pension was £90 per week in December 1992.
In December 1993 it should be

$$\text{pension in December 1992} \times \frac{\text{RPI in December 1993}}{\text{RPI in December 1992}}$$

$$= £90 \times \frac{141.9}{139.2} \simeq £91.75$$

these values are given in Frame 3

2 A pension was £68 per week in January 1993. In January 1994 it should be

$$\text{pension in January 1993} \times \frac{\text{RPI in January 1994}}{\text{RPI in January 1993}}$$

$$= \quad \times \frac{\quad}{\quad} \simeq \quad$$

Frame 9

Exercise 2

An index-linked pension was £104 per week in July 1993.
What should it be in July 1994?

RPI values are given in Frame 6

Frame 10

The purchasing power of the pound

If goods costing £1 now cost 60p four years ago, then we say
'The purchasing power of the pound is 60p compared with four years ago'

The purchasing power (in pence) of the pound at a given date compared with an earlier date is

$$\frac{\text{value of RPI at the earlier date}}{\text{value of RPI at the later date}} \times 100\text{p}$$

Frame 11

Examples

1 The purchasing power (in pence) of the pound in December 1993 compared with a year earlier was

$$\frac{\text{value of RPI in December 1992}}{\text{value of RPI in December 1993}} \times 100\text{p}$$

$$= \frac{139.2}{141.9} \times 100\text{p}$$

$$\simeq 98\text{p}$$

2 The purchasing power of the pound in January 1994 compared with January 1987 was

the base date

$$\frac{\text{value of RPI in January 1987}}{\text{value of RPI in January 1994}} \times 100\text{p}$$

the starting value

$$= \frac{100}{141.3} \times 100\text{p}$$

$$\simeq 71\text{p}$$

Frame 12

Exercise 3

(a) Find the purchasing power of the pound in July 1994 compared with January 1993.

(b) Find the purchasing power of the pound in January 1994 compared with December 1992.

Give your answers to the nearest penny.

RPI values are given in Frames 3 and 6

Frame 6A

Solution to Exercise 1

$$\text{Price ratio} = \frac{\text{value of RPI in July 1994}}{\text{value of RPI in July 1993}} = \frac{144.0}{140.7} \simeq 1.023$$

RPI in July 1994 = 102.3% of RPI in July 1993

The annual rate of inflation in July 1994 was 2.3%.

Frame 9A

Solution to Exercise 2

Pension in July 1994 should be:

$$\text{pension in July 1993} \times \frac{\text{RPI in July 1994}}{\text{RPI in July 1993}} = £104 \times \frac{144.0}{140.7} \simeq £106.44$$

Frame 12A

Solution to Exercise 3

(a) The purchasing power of the pound in July 1994 compared with January 1993 was

$$\frac{\text{value of RPI in January 1993}}{\text{value of RPI in July 1994}} \times 100\text{p} = \frac{137.9}{144.0} \times 100\text{p} \simeq 96\text{p}$$

(b) The purchasing power of the pound in January 1994 compared with December 1992 was

$$\frac{\text{value of RPI in December 1992}}{\text{value of RPI in January 1994}} \times 100\text{p} = \frac{139.2}{141.3} \times 100\text{p} \simeq 99\text{p}$$

6.2 What price yesterday's pound in our pocket?

How cheap (or expensive) was it to live in England in the past? Are people really better off today than in centuries gone by? These are the questions investigated in the reader article 'What price yesterday's pound in our pocket?' This article was first published in the *Independent* newspaper on 26 February 1994.

In the article, the value of a pound in past times was given in terms of how many pounds' worth of goods it would buy in 1994. This measure is the *reciprocal* of the purchasing power of the pound in 1994 compared with the date in the past. (The purchasing power of the pound was defined in Frame 10 of the audiotape sequence on page 63.) For example, £1 in 1984 is equivalent to £1.58 in 1994. So the purchasing power of the pound in 1994 compared with 1984 was:

$$\frac{1}{1.58} \times 100\text{p} \simeq 63\text{p}.$$

Recall, the *reciprocal* of a number is the number one divided by that number, so the reciprocal of 6 is $\frac{1}{6}$, the reciprocal of $\frac{1}{6}$ is $1/\frac{1}{6} = 6$.

Activity 31 The purchasing power of the pound

Read the article 'What price yesterday's pound in our pocket?' to get a sense of the main points it makes. Then answer the following questions.

(a) Use the information given in the article to calculate the purchasing power of the pound in 1994 compared with (i) 1938 (ii) 1914.

(b) 'In the 1530s, the artist Hans Holbein earned ... £7 10s 0d a quarter. That would be £2500 today.' Assuming this claim is correct, calculate the purchasing power of the pound in 1994 (today) compared with the 1530s.
(Before decimalization, there were 12 old pence (d) in one shilling (s) and 20 shillings in one pound, so, for instance, at the time of decimalization (1971) £7 10s 0d became £7.50.)

Activity 32 How did people spend their money?

What percentage of expenditure was spent on food:

(a) during the period from the fifteenth century until the early twentieth century?

(b) in 1994?

What changes in society in England do you think might explain the difference in these percentages?

Activity 33 Reviewing the RPI and inflation

At the beginning of Section 5, you were asked to write down what you thought the RPI was and how it might be used to measure inflation. Now that you have read quite a lot about the RPI and its uses, look back at what you wrote then. Make a note of which of the ideas that you had were accurate, and note any points that you now realize need modification. If you have not done so already, write a brief description in your own words of what the RPI measures, what is meant by the annual rate of inflation, and how the RPI is used to calculate the annual rate of inflation. Use your Handbook activity sheet, and add to your existing notes where necessary.

Pause for a few minutes to review the notes you have made on prices and price changes as you have worked through this unit. Check that you have made notes on any topics or ideas that you feel would be useful. Is there anything you wish to add to your notes? If so, why not do it now while the ideas are still fresh in your mind? Also, think about *how* you have been reading for learning using the text, and what has been particularly helpful to you.

In this section, you have seen how price rises are measured using an index of retail prices. Earnings are discussed in the next unit. Only when prices and earnings have both been considered can you begin to answer the central question of these two units: 'Are people getting better off?' In the final section of the next unit, you will see how to use the RPI in conjunction with an index of earnings to see whether rises in earnings are keeping pace with rises in prices. Before turning to earnings, however, we briefly review some of the mathematical ideas that you have met in this unit.

Outcomes

After studying this section, you should be able to:

- ◇ use the RPI to calculate the annual rate of inflation (Frames 5, 6);
- ◇ use the RPI to calculate the value of an index-linked pension (Frames 8, 9);
- ◇ use the RPI to calculate the purchasing power of the pound at one date compared with another (Frame 12);
- ◇ extract and interpret information from a newspaper article about the cost of living (Activities 31, 32);
- ◇ explain to someone else how the RPI is related to the current purchasing power of the pound (Activity 33).

7 Some mathematical themes

Aims The main aim of this section is to review some of the mathematical skills and ideas you have been using, and for you to reflect on some of their more general features and applications. ◇

7.1 Relative and absolute comparisons

The distinction between relative and absolute comparisons is an important one that has run through this unit and also came up in *Unit 1*. Here, its meaning and significance will be made more explicit. You will be shown examples which illustrate the difference between absolute and relative measures and you will be asked to reflect on why calculating in relative terms is often a better way to make a fair comparison.

Start with a simple example based on comparison of births between countries. In 1992, there were roughly 50 000 babies born in Ireland and 800 000 born in the UK. So, in absolute terms, there are many more births in the UK (800 000 is far more than 50 000), but so what? The population of the UK is much greater than that of the Republic of Ireland, so this difference is not unexpected. A more useful and interesting comparison is the birth *rates* of the two countries, and for this, additional data are required.

Table 15 Births and population of the Republic of Ireland and the UK, 1992

	Births (est.)	Population (million)
Republic of Ireland	50 000	3.5
UK	800 000	57.7

The birth rate is normally calculated as the number of births per 1000 of the population.

The birth rate for the Republic of Ireland in 1992 was

$$\frac{50\,000}{3\,500\,000} \times 1000 = 14 \text{ (rounded to the nearest whole number).}$$

Activity 34 Comparing birth rates

(a) Use Table 15 to calculate the UK birth rate. Give your answer to the nearest whole number.

(b) How do the birth rates for the Republic of Ireland and the UK compare?

The point of this activity has been to re-emphasize that absolute comparisons are often not very helpful, and that the calculation of a

proportion or rate (that is, a relative comparison) is usually more meaningful. Here is another activity designed to reinforce this point.

Activity 35 *Massaging the figures*

1 billion = 10^9

Between 1981 and 1992, UK government spending on 'Housing and community amenities' rose from £7.1 billion to £10.9 billion. Over the same period, total government expenditure rose from £117.1 billion to £254.1 billion. (Source: *Social Trends 24*, page 91, Table 6.21)

(a) Use the data above to make a case that the government has done well in its provision of 'Housing and community amenities' over this period.

(b) Use the data above to make a case that the government has done badly in its provision of 'Housing and community amenities' over this period.

Pie charts offer another kind of graphical image. You could add to the notes you made for Activity 8.

The distinction between absolute and relative difference can be represented graphically, as follows.

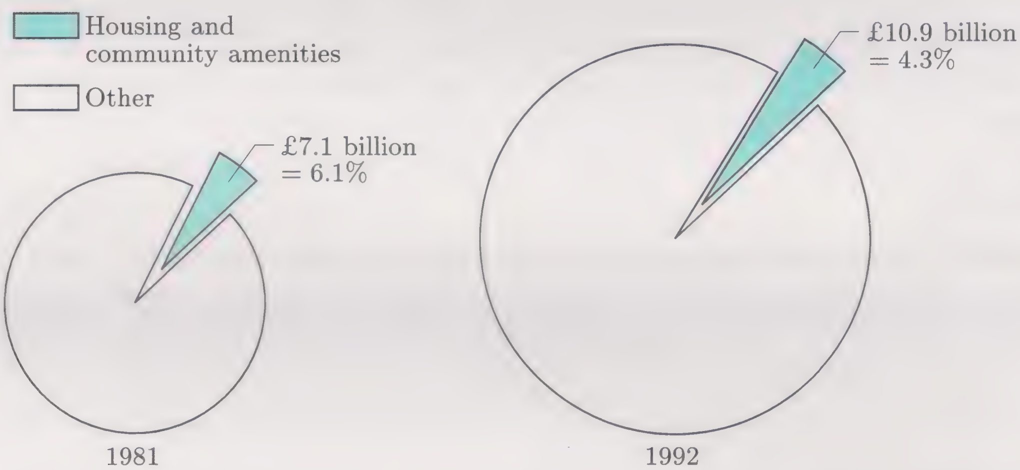


Figure 5 Pie charts showing UK government spending on 'Housing and community amenities' as a percentage of total government expenditure in 1981 and 1992

(Note that the circles have been drawn so that their areas are in proportion to the total government spending for each year.)

As the diagram shows, *in absolute terms*, there is more 'cake' in the shaded slice for 1992 than for 1981 (spending has gone up from £7.1 billion to £10.9 billion). But *relatively speaking*, the 1992 slice is smaller—it has fallen from 6.1% to 4.3% of the overall cake.

The strength of using relative measures, such as percentages, is that they take account of the size of the base from which the measure is taken. However, there is a weakness in this approach as well, in that there may be a loss of valuable information. For example, an American newspaper once claimed that a survey had shown 60% of the electorate to be in favour of a

particular candidate. What the journalist did not reveal at the time was that his 'survey' consisted of five men in a bar (whom he had asked the previous evening), three of whom had expressed a preference for the candidate in question. Similarly, if someone earning £5000 a year and someone earning £50 000 a year both get a 5% pay rise, is this 'fair'?

Equal percentage rises widen absolute differences.

Nevertheless, although percentages too may not always have the desired effect or be misleading, overall, expressing measures as ratios (one number divided by another) is a powerful idea in mathematics across a range of mathematical concepts.

7.2 Ratio and proportion

It is easy to distinguish children from adults. For one thing, children are usually much smaller. But how are we able to tell them apart from a drawing alone? Have a look at the two outline drawings. Which one do you think represents the child and which the adult?

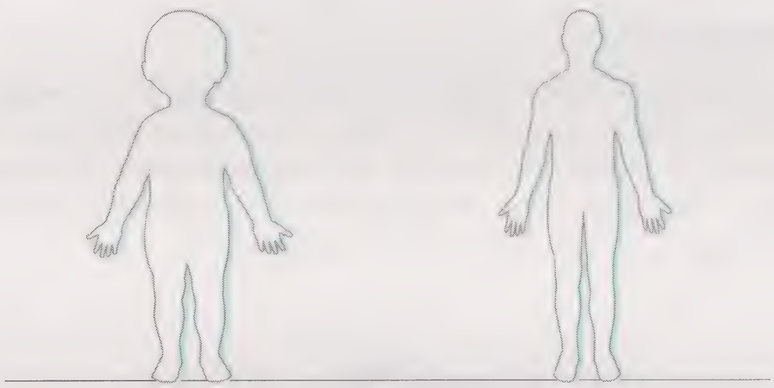


Figure 6 Two outline drawings

Although the drawings are the same size, the proportions are different. The left-hand drawing, which represents the child, has a larger head in relation to the size of the body. The differences between adults and children in this respect are quite dramatic, as the particular examples in Table 16 show. It is not just to do with skin texture, dress or body posture. Children are actually a different shape from adults.

Table 16 Heights and head circumferences of children and adults

Name	Age	Height (cm)	Head circumference (cm)	H/C
David	1 day	51	35.5	
Shelley	2 years	82	49	
Lydia	5 years	114	52.5	
Ruth	10 years	127.5	54.5	
Bal	12 years	144	54.5	
Marti	33 years	157.5	55.5	
Sunil	37 years	172	55	

Activity 36 Comparing height/circumference ratios between adults and children

(a) For each of the seven people in Table 16, calculate (with the help of your calculator) their height (H) divided by their head circumference (C). Write your results in the empty column of the table headed H/C.

What do these results suggest?

(b) If you can, take the same measurements from members of your own or a friend's household, and check that the same pattern is true for them.

You may not have been consciously aware that adults and children differ so dramatically in terms of their body proportions. Perhaps one of the reasons that people have difficulty with the topic of proportion in mathematics is that we tend to exercise our 'sense of proportion' at an instinctive or subconscious level. Are you clear about what the word 'proportion' really means?

What is proportion?

A common criticism of many children's and some adult's drawings is that certain parts are not 'in proportion'. That means that they are either too big or too small *in relation to* the rest of the masterpiece. 'In proportion' means being in the same ratio. Imagine that you have drawn a picture of your house, reducing it in scale to one twentieth of its size.

If your drawing is to be 'in proportion', then every length detail must be drawn $\frac{1}{20}$ of the original size. So if the door of your house is 2 m high, the door in your drawing should be 10 cm high if it is to be 'in proportion'. In other words, if you take *any* length measurement from your house and divide it by the corresponding measurement for your drawing, the answer should be exactly twenty.

The numerical answer that you get when you divide one measurement by another is called the *ratio* of their measurements.

Activity 37 The camera does not lie

Table 17 contains some actual body measurements along with the corresponding measurements taken from a photograph. If you have a photograph of yourself, you may care to use your own figures here.

Table 17 Ratios of person and photograph

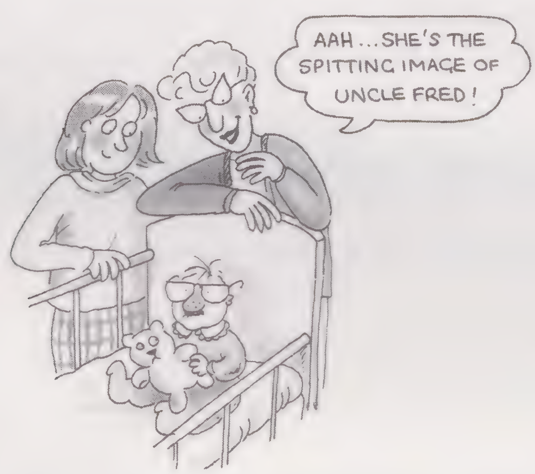
	Actual person M (cm)	Photograph P (cm)	Person/photograph ratio M/P
Height	173	4.1	
Shoulder width	44.5	1.1	
Arm length	71	1.7	
Foot length	25	0.6	

(a) Calculate the actual ratios of the measurements to the corresponding measurements taken on the photograph and record them in the final column of the table. (Again, use a calculator.)

(b) What can you say about the size of the photograph?

Two shapes which are in proportion to each other have the same shape. In mathematics, they are said to be *similar*. The word 'similar' is used rather precisely in mathematics, in contrast to how it is often used in everyday language. For example, a teacher might be unhappy that two exam scripts look so 'similar'. Two sisters or brothers might look 'similar'.

Recall the discussion of this word in 'Communicating mathematically' on the preparatory audiotape.



Here the word means simply 'alike in certain respects'. Just what exactly it is that makes them alike is often not made clear. In mathematics, however, the word 'similar' means having the same *shape*.

The two triangles drawn in Figure 7 are said to be similar, because one is an exact scaled-up version of the other. Because they are the same shape, the corresponding (matching) angles are equal and the corresponding sides are in proportion (that is, in this case, the sides in the larger triangle are each twice as long as those of the smaller).

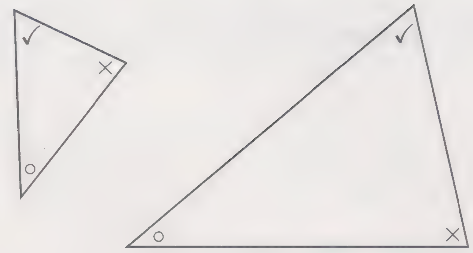


Figure 7 Two similar triangles?

As you can see in Figure 8, when the smaller triangle is rotated and placed inside the larger one, it becomes obvious that they have the same shape.

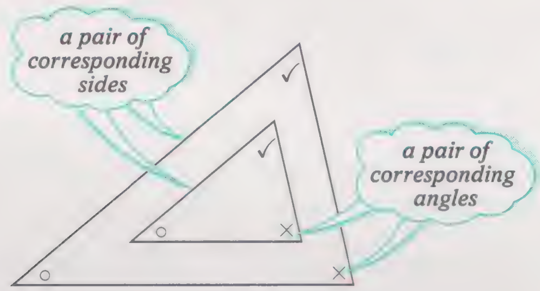


Figure 8 Two similar triangles!

This redrawn form of representation allows the matching up of the sides and the angles of the two triangles and observe which ones *correspond* with each other. Matching up these corresponding components reveals two important properties of similar shapes. First, their corresponding angles are equal (the angles marked × are equal, and so on). Second, their corresponding sides are in the same proportion—in this case, the ratio of larger to smaller is constantly two to one. This ratio is often written as 2:1 (and read 'two to one' or 'two is to one' or 'the ratio two to one').

Activity 38 Similar, mathematically

Which of the following pairs of shapes are similar (in the mathematical sense)?

(a) Any two squares.

(b) Any two rectangles.

(c) Any two circles.

(d) Any two equilateral triangles (an equilateral triangle is one with all three sides equal in length and all three angles equal in size).

(e) Any two right-angled triangles.

The connection between scale and maps will be explored in *Unit* 6.

An idea which is helpful in all problems on proportion is that of a *scale factor*. In Figures 7 and 8, the scale factor was the number multiplied by, which was two. In the example of the photograph (Activity 37), the scale factor was about one-fortieth.

Now for a more practical example of proportion, try scaling the ingredients of the following recipe.

Activity 39 More ice cream

The ingredients for six servings of hazelnut ice cream are given below. Complete the table for eight servings.

Ingredients	Amounts for six servings	Amounts for eight servings
Toasted hazelnuts	225 g	
Cornflour	2 tablespoons	
Separated eggs	2	
Castor sugar	75 g	
Milk	300 ml	
Vanilla essence	a few drops	
Double cream	300 ml	

The ratio 'amount for eight' : 'amount for six' is generally the same for each ingredient. In general, the concepts of ratio and scale factor are essentially the same and indeed are used in the notion of price ratio which you met in Section 4. These three ideas are all to do with things getting bigger and smaller and the sort of calculations which are used to describe such changes *proportionately*. It might be a good idea to spend a few minutes now thinking about the ideas of ratio, and price ratio, and how they are all built around the same mathematical idea, namely multiplication (and its inverse operation, division).

Add to your Handbook activity sheet.

7.3 Is a picture worth a thousand words?

This final subsection is an overview of the various modes of communication used so far—words, tables and graphs, and diagrams among others. You may have a preference for one over the others as a way of presenting ideas and of receiving information. But, as was mentioned in *Unit 1*, you need to learn how to 'read' all these different structured images. They can all aid your understanding and communication of different mathematical ideas. You need to develop your skills in using and interpreting all of them to make the most of MU120.

Look back at Figure 4 on page 49, which shows the structure of the RPI. Notice how the slices of the central circle show by their size how large each group is compared with the others. For example, you can see at a glance that the slice representing 'Food and catering' is roughly twice the size of the slice representing the group 'Alcohol and tobacco'. Other features are equally easy to pick out; for example, 'Housing and household expenditure' is visually the largest slice and 'Personal expenditure' the smallest. None of these features would be immediately apparent from the numbers alone. (If you are not convinced of this, turn to the weights in Table 13 on page 50 and see if these patterns leap out from the numbers as powerfully as they do from the graphical representation in Figure 4.)

Activity 40 Reviewing various modes of communication

Look back through the notes which you have made about diagrams and tables (starting with Activity 8). Pick out at least one example each where information has been communicated:

(a) in words;

(b) in a table;

(c) in a graph or diagram.

Note down at least one strength and one weakness of each approach. Give an example when it would be helpful to use a table, and when a graph. Try to use more than one of these modes of communication in your notes.

Finally, think about whether or not there are any further modes of communication (recall the various ways of seeing mathematically discussed in *Unit 1*) that have not been included here, but which may be useful in other contexts.

Use the printed response sheet that you used first in Activity 8.

Outcomes

After studying this section, you should:

◇ understand the distinction between relative and absolute comparisons (Activities 34, 35);

◇ understand the terms *ratio* and *proportion* and be able to use them appropriately (Activities 36, 37, 38, 39);

◇ be able to identify the strengths and weaknesses of using words, tables and graphs or diagrams to present information (Activity 40).

Unit summary and outcomes

This unit has looked at a variety of ways of comparing prices, and the construction of a price index. Important statistical ideas that contributed to this included mean, weighted mean and median, as well as the general notion of an index.

Activity 41 Thinking about your progress

You now know quite a lot about the RPI, and so you should be able to explain what politicians and journalists really mean when they make sweeping statements about inflation and the cost of living. In the course of discussing this topic several mathematical ideas have been introduced; we suggested that you record some of these, such as weighted means and the relation between graphical and tabular methods of representing data, in your Learning File. Now would be a good time to reflect for a few minutes on your progress so far.

Think about what you knew at the beginning of the unit, and compare it with what you know now. How have you gone about using the text for your learning and what has particularly helped you (for example, calculator work, audiotape sequences)? What topics in this unit have you found straightforward? What have you found difficult?

Finally, write down an example of something that caused you difficulty and on which you feel you need to spend more time. And, if you have identified some aspect of the work in this unit that is causing you real concern, how might you go about overcoming this? What sources of assistance are available to you? Is your difficulty something that you could sort out by referring to a particular section of the preparatory materials? Would it help to talk it over with other students? Why not ask your tutor for extra explanation if there is something that you did not understand?

Look back to your planning sheet for *Unit 2*. The changes you planned to make (if any): were you able to make them? What effect did they have on how you worked? How do you intend to approach the next unit (which is also on statistical ideas)? Decide now what action you plan to take: write it down in your Learning File. And when you have carried out your plan—tomorrow, or next week, or whenever—remember to record in your file the results of your positive action and your feelings about them. Make a note for future reference of what helps you personally and what does not.

There is a printed response sheets for this activity.

Outcomes

After studying this unit, you should be able to:

- ◇ calculate the mean, weighted mean and the median of a batch of numbers;
- ◇ calculate a percentage and a percentage price increase;
- ◇ use a weighted mean to find an average percentage price increase (given the weights);
- ◇ use the statistical facilities of your calculator to find the mean, the median and (given the weights) the weighted mean of a batch of numbers;
- ◇ calculate a percentage price increase and a price ratio from a price index;
- ◇ use price ratios to work out values of a price index;
- ◇ calculate the RPI given the relevant information;
- ◇ describe the likely effect of particular price changes on the value of the RPI;
- ◇ use the RPI to calculate the annual rate of inflation, the value of an index-linked pension and the purchasing power of the pound at one date compared with another;
- ◇ extract and interpret information from an article about the cost of living;
- ◇ read and interpret data from a table, a graph or a diagram;
- ◇ understand the distinction between relative and absolute comparisons;
- ◇ understand the terms 'ratio' and 'proportion' and use them appropriately;
- ◇ discuss the strengths and weaknesses of using words, tables, graphs and diagrams to present information;
- ◇ explain a familiar mathematical idea in your own words;
- ◇ understand the meaning of expressions such as $\sum x$ and $\sum xw$ and use them.

You should also be able to use your calculator to:

- ◇ clear existing data lists;
- ◇ enter data into lists and edit the data;
- ◇ find the median and the mean of a batch of data;
- ◇ find the mean of a batch of frequency data;
- ◇ produce simple sequences of numbers.

Comments on Activities

Activity 1

See the comments after the activity.

Activity 2

See the comments after the activity.

Activity 3

See the comments after the activity.

Activity 4

The cost of a loaf today as a percentage of a typical daily wage is:

$$\frac{56}{3600} \times 100\% \simeq 1.6\%.$$

This is rather less than in 1594, suggesting that people are better off now.

Activity 5

See the comments after the activity.

Activity 6

(a) Assuming that quotations for all five types of loaf were obtained from each shop whenever possible, roughly 350 shops were surveyed. The number varies slightly from one type of loaf to another, presumably because not all shops stocked all these types of loaf. We are not told, however, that the same shops were used to price different loaves, but it seems a reasonable assumption.

(b) The cheapest type of large loaf, if we go by average prices, is the white sliced variety. However, given the variation in the quotations (shown in the final column of the table), you could actually pay more for a large white sliced loaf (for example, 76p) than for the 800 g brown unsliced loaf (some of these cost as little as 73p). Notice, however, that the price range quoted in the final column of the table gives only the band within which 80% of the prices fell, presumably the middle 80% band of prices.

(c) The lowest price quoted here for the small brown loaf is 41p. However, note that the price ranges given in the final column of the table are those within which 80% of the quotations fell. It is quite likely, therefore, that there were some small brown loaves in the survey that were being sold for less than 41p (as well as some being sold for more than 56p).

(d) The greatest variation in price seems to occur in the large white sliced loaf—in fact, the cheapest price quoted here for this type of loaf, 39p, is about half the cost of the most expensive, 76p. The two narrowest ranges are for the two unsliced loaves. However, because you only know about the middle 80% of prices, there is uncertainty about the very extreme prices for these loaves. The reason for the wide range in price of the large white sliced loaf may be due to its popularity. This may result in very competitive pricing of the large white sliced loaf in many supermarkets, thereby extending the range of prices downwards.

(e) Tables can make information easy to find, and to read, if they are clearly laid out and well-labelled. They are compact, yet contain the 'real' numbers. Tables help to structure the information, by selecting key features for the horizontal and vertical elements. When information has only been presented in table form, it can sometimes be hard to 'un-table' it in your mind, in order to re-present it in some other form—including another table organized along different lines.

Activity 7

(a) Refer back to the preparatory material if you are unsure.

(b) Adjacent points on the graph refer to the February prices in successive years of a large white sliced loaf. Joining two adjacent points allows you to make a reasonable guess at the bread prices during the intervening months. The procedure of joining up the dots is valid here because the prices are being compared over time and time is measured on a continuous scale. With certain other sorts of graph it would not be legitimate. For example, in *Unit 5* you will see bird populations which fluctuate widely over the course of the year.

(c) The steepest upward-sloping parts of the graph are from 1980 to 1981, 1988 to 1989 and from 1990 to 1991. There is also a fairly steep downturn from 1993 to 1994. These sections show that the price of bread changed more in a single year than during other years between 1980 and 1994).

(d) There is no comment for this part.

Activity 8

When presenting results and data it is important to think about the most appropriate method for displaying the work for ease of use. Thinking about and discussing the advantages and disadvantages of different methods helps you to become more critical in displaying and presenting your own data.

Activity 9

(a) Bread prices went up by 1p (from 54p to 55p) between February 1992 and February 1993. Expressed as a percentage of the February 1992 price, this is

$$\frac{1}{54} \times 100\% = 1.9\% \quad \text{(rounded to one decimal place).}$$

(b) The first increase (from February 1981 to February 1982) was 2.8%.
The second increase (from February 1992 to February 1993) was 1.9%.

When expressed in percentage terms, the first price increase was about *one and a half* times as much as the second one.

(c) Bread prices went up by 23p (from 32p to 55p) between February 1980 and February 1993. Expressed as a percentage of the February 1980 price, this is

$$\frac{23}{32} \times 100\% = 71.9\% \quad \text{(rounded to one decimal place).}$$

Activity 10

(a) Average weekly earnings went up by £162.6 (from £111.7 to £274.3) between 1980 and 1993. Expressed as a percentage of 1980 earnings, this is

$$\frac{162.6}{111.7} \times 100\% = 145.6\% \quad \text{(rounded to one decimal place).}$$

This is an increase substantially over 100%, or, in other words, average weekly earnings have more than doubled.

(b) Average male earnings have gone up much more than the average price of bread. See also the comments in the text following this activity.

Activity 11

See the comments after the activity.

Activity 12

See the comments after the activity.

Activity 13

See the comments after the activity.

Activity 14

The mean weekly pocket money is

$$\begin{aligned}&\pounds(2.00 + 2.50 + 2.50 + 3.00 + 10.00)/5\\&\quad = \pounds 4.00.\end{aligned}$$

The median is the middle value: £2.50.

See also the comments in the text following the activity.

Activity 15

Check your definitions against those on pages 24 and 25.

Here are some developmental testing students' comments.

How I learned mean and median

'For part of this study session I aimed to know what the mean and median are and how to calculate them. I feel confident that I can do this now as I've just finished working through Section 2, and feel I understand the terms.'

How I learned the terms

'I worked through the explanation of mean and stopped after the worked example. I sensed I was understanding the term "mean" as I'm used to working out averages in this way. I carried on and read through the definition of "median" and stopped after Example 2. I read through this example again, as I think I missed the point the first time. I carried on reading and then did Activity 14. I had no trouble working this out.

I repeated each separate definition mentally to myself once or twice to test myself. Something told me I will remember these definitions for the next session—but I also feel sure I will remember them better when I have used them more.'

The second description above demonstrates how this student has gone about learning the terms 'mean' and 'median'. The strategies she has used include focusing on a particular aspect while reading, checking the meaning of each of the terms by doing examples and activities, repeating meanings for self-assessment, and so on. Sometimes it is useful to pause and think how you are going to approach learning a particular part of the course—for example, it may be unwise to try to learn how to use the calculator effectively by reading alone. Already you have a wide range of learning strategies to use—reading for learning, repeating to yourself, writing something down, using tables and graphs, using an audiotape and a videotape, using a calculator—to name just a few! Different strategies can be used to learn different things. Students who are aware of how they learn tend to learn both more effectively and more efficiently. This will not happen overnight and some activities may help you to do this better.

Activity 16

There is no comment for this activity.

Activity 17

There is no comment for this activity.

Activity 18

If the percentage price increases are labelled p and the weights w, then the weighted mean of the percentage price increases is

$$\frac{\sum pw}{\sum w} = \frac{41668}{478} = 87\% \text{ (rounded to the nearest whole number).}$$

So the weighted mean is 87%.

Activity 19

In this case, $\sum pw = 416680$ and $\sum w = 4780$, so the weighted mean is

$$\frac{\sum pw}{\sum w} = \frac{416680}{4780} = 87 \text{ (rounded to the nearest whole number).}$$

This is the same value as that obtained in Activity 18.

One student commented: 'I didn't believe it could be the same, so I was surprised when it was. I wondered whether there was something special about the ten, so I did the same activity

again using five. It still came out the same. As I did the calculation again, I began to get a better feel for what was involved.'

Activity 20

(a) The percentage price rise is given by

$$\begin{aligned}\frac{\text{price rise}}{\text{original price}} \times 100\% &= \frac{39-30}{30} \times 100\% \\ &= \frac{9}{30} \times 100\% \\ &= 30\%.\end{aligned}$$

(b) The price ratio is

$$\frac{\text{final price}}{\text{original price}} = \frac{39}{30} = 1.30.$$

(c) The price ratio is so called because it is the ratio of two prices.

(d) The rules for converting a percentage price rise into a price ratio and vice versa may be written as follows.

$$\text{price ratio} = 1 + \frac{\text{percentage price rise}}{100}$$

$$\begin{aligned}&\text{percentage price rise} \\ &\qquad = (\text{price ratio} - 1) \times 100\%\end{aligned}$$

Activity 21

(a) The price ratios are given in the table below.

Item	Price ratio (1994 price ÷ 1980 price)	Average 1980 weekly bill (pence)
Large w. loaf	1.47	165
Milk	2.12	223
Eggs	1.89	31
Potatoes	2.10	46
Sugar	1.86	13
Total		478

(b) If the price ratios are labelled r and the weights w, then $\sum rw = 894.68$ and $\sum w = 478$. So the weighted mean of the price ratios is:

$$\frac{\sum rw}{\sum w} = \frac{894.68}{478} \simeq 1.87.$$

Exercise in Tape Frame 6

(a) The price ratio for 1988 relative to 1987 was $104.5/100 = 1.045$. So the average price of a large white sliced loaf rose by 4.5% between January 1987 and January 1988.

(b) The price ratio for 1990 relative to 1987 was $113.6/100 = 1.136$. So the average price of a large white sliced loaf rose by 13.6% between January 1987 and January 1990.

(c) The price ratio for 1993 relative to 1990 was $125.0/113.6 = 1.100$. So the average price of a large white sliced loaf rose by 10.0% between January 1990 and January 1993.

Activity 22

If the price ratios are called r and the weights w then $\sum rw = 604.7$ and $\sum w = 580$. So the weighted mean of the price ratios is

$$\frac{\sum rw}{\sum w} = \frac{604.7}{580} \simeq 1.043.$$

So the price ratio for the basket for January 1988 relative to January 1987 was 1.043.

Activity 23

The values of the price index are given in the table below.

Year	Value of index in January
1990	$109.5 \times 1.065 \simeq 116.6$
1991	$116.6 \times 1.070 \simeq 124.8$
1992	$124.8 \times 1.038 \simeq 129.5$
1993	$129.5 \times 1.005 \simeq 130.1$
1994	$130.1 \times 1.016 \simeq 132.2$

Activity 24

(a) The price ratio for January 1994 relative to January 1990 is given by

$$\begin{aligned}\frac{\text{value of index in January 1994}}{\text{value of index in January 1990}} &= \frac{132.2}{116.6} \\ &\simeq 1.134.\end{aligned}$$

So the price of the basket rose by 13.4% between January 1990 and January 1994.

(b) The price ratio for January 1992 relative to January 1989 is given by

$$\frac{\text{value of index in January 1992}}{\text{value of index in January 1989}} = \frac{129.5}{109.5} \simeq 1.183.$$

So the price of the basket rose by 18.3% between January 1989 and January 1992.

Activity 25

There are no comments on this activity.

Activity 26

(a) What you need to remember here is that the size of an area represents the proportion of expenditure on that class of goods or services.

(i) The sector for 'Personal expenditure' looks as if it is approximately a tenth of the whole inner circle—so approximately a tenth of total weekly expenditure is personal expenditure.

(ii) 'Housing and household expenditure' looks as if it is approximately one third of the inner circle, so approximately a third of expenditure is on housing and household expenditure.

(iii) The area for 'Housing' takes up about a seventh of the outer ring, so about a seventh of expenditure is on housing.

(b) (i) The amount spent each week on 'Personal expenditure' is approximately

$$\frac{1}{10} \times £240 = £24.$$

(ii) The amount spent each week on 'Housing and household expenditure' is approximately

$$\frac{1}{3} \times £240 = £80.$$

(iii) The amount spent each week on 'Housing' is approximately

$$\frac{1}{7} \times £240 \simeq £34.$$

Recall, however, that the weights represent *average* proportions of expenditure, so these estimates will only be good ones if the spending patterns of this household are similar to those of the 'typical' household.

Activity 27

There are no comments on this activity.

Activity 28

(a) The basket of goods used in the construction of the RPI represents the way that a typical index household spends its money. All households except high-income households and those of pensioners are classed as index households.

(b) The weights of the items in the basket reflect the spending patterns of households; the weight of an individual item represents the proportion of expenditure on that item.

(c) The contents of the basket are changed from time to time to ensure that all the items still feature in a typical household's shopping basket, and to allow for new items to be included as they become important items of expenditure.

(d) Data collected in the Family Expenditure Survey are used to obtain estimates of the average expenditure (of an index household) on each item in the index; these figures are used as weights (after being adjusted so that their sum is 1000).

Activity 29

There is no comment for this activity.

Exercise in Tape Frame 5

March 1994

All-item price ratio = 1.008697

$$\begin{aligned}\text{Value of RPI in March 1994} &= 141.3 \times 1.008697\\ &= 142.5288861\\ &\simeq 142.5.\end{aligned}$$

June 1994

All-item price ratio = 1.023864

$$\begin{aligned}\text{Value of RPI in June 1994} &= 141.3 \times 1.023864\\ &= 144.6719832\\ &\simeq 144.7.\end{aligned}$$

Activity 30

More detail has been included in this comment than we expect you will have given in your answer. When you have read it, make sure you understand all the points we have included. If your explanations were much shorter, ask yourself whether your explanations were really sufficient. If you think not, note particularly any additional points that you did not include.

(a) The RPI is calculated using the price ratio and weight of each item. Since the weights of items change very little from one year to the next, the price ratio alone will normally tell you whether a change in price is likely to lead to an increase or a decrease in the value of the RPI. If a price rises, then the price ratio is greater than one, so the RPI is likely to increase as a result. If a price falls, then the price ratio is less than one, so the RPI is likely to decrease. Therefore, since the price of food rose, this is likely to lead to an increase in the value of the RPI. But since the price of furniture fell, this is likely to lead to a decrease in the value of the RPI.

(b) Both changes are likely to be small for two reasons. First, the price changes are themselves small. Second, food and furniture form only part of a household's expenditure: no single group, subgroup or section will have a large effect on the RPI on its own, unless there is a very large change in its price.

(c) The weight of 'Food' was 142 in 1994. Since 'Furniture' is only one section in the subgroup 'Household goods' which had weight 76 in 1994, the weight of 'Furniture' is much smaller than 76. So the weight of 'Food' is much larger than the weight of 'Furniture'.

(d) Since the weight of 'Food' is much larger than the weight of 'Furniture', and the percentage change in the prices are similar in size, the change in the price of food is likely to have a much larger effect on the value of the RPI as a whole.

(e) Strategies included: doing calculations indicated in the activities to get more of a sense of how it might go 'in general'; writing down ideas and earlier understandings; reading and re-reading the text.

Activity 31

(a) (i) The 1938 pound was equivalent to £32 in 1994, so the purchasing power of the pound in 1994 compared with 1938 was:

$$\frac{1}{32} \times 100\text{p} \simeq 3\text{p}.$$

(ii) The 1914 pound was equivalent to £50.20 in 1994, so the purchasing power of the pound in 1994 compared with 1914 was:

$$\frac{1}{50.2} \times 100\text{p} \simeq 2\text{p}.$$

(b) The purchasing power of the pound in 1994 compared with the 1530s was:

$$\frac{7.5}{2500} \times 100\text{p} \simeq 0.3\text{p}.$$

Activity 32

(a) 80%

(b) 14.4%

Most people in England are no longer living in a subsistence economy. Most people's income is much more than is needed for basic survival, and a huge diversity of goods is available for them to spend their money on. Thus, a much smaller proportion of expenditure is spent on one basic requirement for survival: food. Though housing is increasingly becoming another matter.

Activity 33

There is no comment for this activity.

Activity 34

(a) In 1992, the birth rate for the UK was

$$\frac{800\,000}{57\,700\,000} \times 1000$$
$$= 14 \text{ (rounded to the nearest whole number).}$$

(b) The two birth rates are roughly the same.

Activity 35

This response is a much fuller answer than you would have been expected to produce.

(a) This first 'newspaper cutting' praises government performance. Note that it does so by ignoring total government spending and inflation over the period in question.

Massive increase of 54% in just eleven years

Government spending on Housing and community amenities rose from £7.1 billion to £10.9 billion, a massive increase of £3.8 billion, or 54% in just eleven years. This figure of £10.9 billion represents a spending of £188.91 for each man woman and child throughout the length and breadth of the UK.

(b) This second 'newspaper cutting' criticizes government performance. Note that it does so by ignoring the absolute increase in government spending on housing over the period in question.

Chancellor, can I have my eighty quid back please?

In 1981, out of every £1 it spent, the government spent a miserly 6.1 pence on Housing and community amenities. By 1992, this figure had fallen to 4.3 pence, a drop of 30%. Based on 1992 government spending figures, this represents a loss to the public of a massive £4.6 billion, or £80 to every man, woman and child up and down the country. So, please, Mr Chancellor, can I have my £80 back?

Activity 36

(a) The ratios in the table below are given to two decimal places.

Name	Age	H/C
David	1 day	1.44
Shelley	2 years	1.67
Lydia	5 years	2.17
Ruth	10 years	2.34
Bal	12 years	2.64
Marti	33 years	2.84
Sunil	37 years	3.13

For most adults, their height is about three times their head circumference. However, with very young children, their height is only about one and a half times their head circumference.

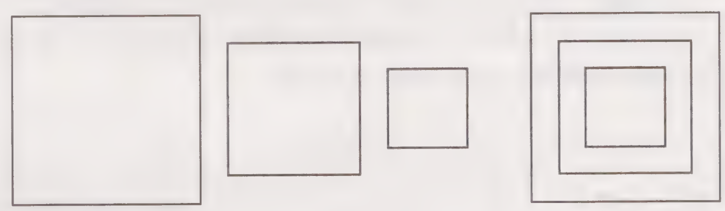
Activity 37

(a) The four ratios are, roughly, 42.2, 40.5, 41.8 and 41.7. So the ratio M/P is about 40 : 1.

(b) The photograph is about $\frac{1}{40}$ of life size.

Activity 38

(a) Yes; as you can see from the diagram below, all squares have the same shape.

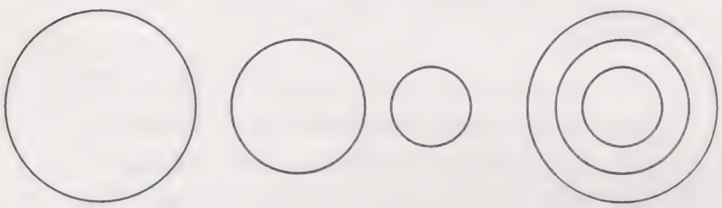


(b) No; two rectangles may have the same shape, but not necessarily. They will be similar only if the ratio of length to breadth is the same for both rectangles. For example, rectangles A and B overleaf are

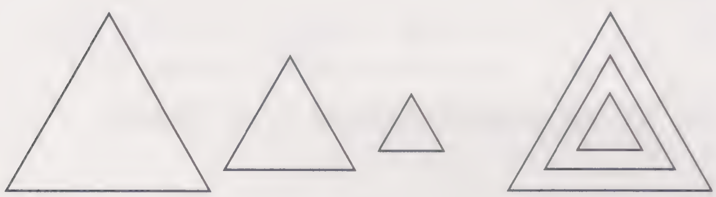
similar, but rectangle C is not similar to either A or B.

A B C

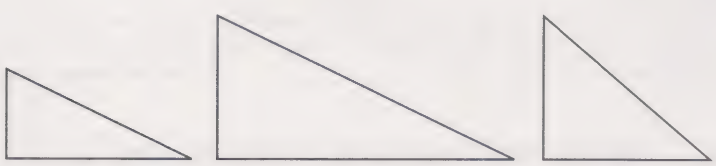
(c) Yes; as demonstrated in the diagram below, all circles have the same shape.



(d) Yes; as you can see below, all equilateral triangles have the same shape.



(e) No; two right-angled triangles may have the same shape, but not necessarily. They will only be similar if all the corresponding angles are the same. For example, triangles A and B below are similar, but triangle C is not similar to either A or B.



A B C

Activity 39

There is no single correct way of tackling this problem. One approach is to say that eight is one-third more than six, so multiply each number in the second column by a scale factor $1\frac{1}{3}$. In some cases, this is fairly straightforward. For example:

Hazelnuts	$225\,\text{g} \times 1\frac{1}{3}$	$= 300\,\text{g}$
Castor sugar	$75\,\text{g} \times 1\frac{1}{3}$	$= 100\,\text{g}$
Milk	$300\,\text{g} \times 1\frac{1}{3}$	$= 400\,\text{g}$

However, others were less obvious. It is difficult to crack $2\frac{2}{3}$ eggs for example! A solution would be to use three small eggs and three tablespoons of cornflour (perhaps ensuring that the third spoon would not be quite full). Of course, not every measure in a recipe needs to be scaled by exactly the same scale factor. There are areas where the unthinking application of proportion breaks down (for example, cooking time and oven temperature) and common sense and practical experience take over. (There are also jokes like: 'Henry VIII had six wives in his lifetime; how many wives did Henry VI have?' or (from a pre-compact disc and pre-metric era) 'If a $33\frac{1}{3}$ rpm record is 12 inches across, how big is a 45 rpm record?')

Activity 40

Words are usually familiar and comfortable. They can be good for communicating subtlety and shades of meaning in aspects of description which are difficult to quantify. For example, describing someone's mood is not something which can easily be expressed in numbers; words alone or together with images seem best.

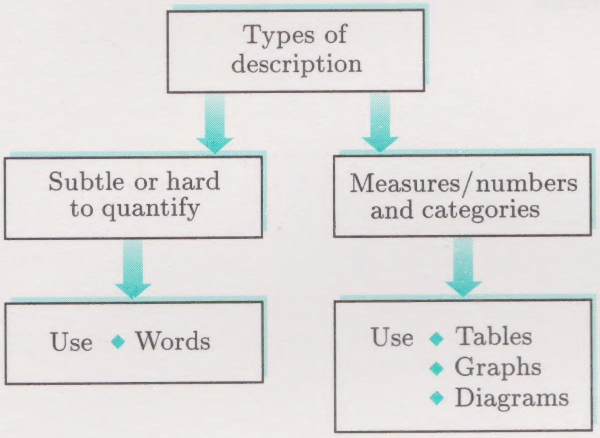
Tables emphasize information which exists in the form of numbers and categories. The individual slots generally hold the numbers and the row and column headings show the categories. Tables are good at providing detail (exact values), but not so good at showing an overview (which value is largest, smallest, and so on).

Graphs and diagrams, like tables, operate on categories and numerical information. However, they are complementary to tables in that they emphasize overall patterns and trends at the expense of providing detail in the form of exact values. Graphs use size or position to represent

numbers whereas diagrams tend to be used to represent relationships. This means that the making of relative comparisons is easy. For example:

- ◇ one bar looks roughly two-thirds the height of another;
- ◇ one bar is the tallest/shortest;
- ◇ there is a wide spread/narrow spread of values, etc.

These ideas are summarized diagrammatically below.



	Overview	Detail
Tables	Weaker	Stronger
Graphs	Stronger	Weaker

The final task of the activity was to consider any further modes of communication not included here. An important mode in mathematics that you may have thought of is the use of *symbols* and these will be looked at in some detail later in the course, particularly in *Unit 8*.

Activity 41

There is no comment for this activity.

Acknowledgements

Grateful acknowledgement is made to the following sources for permission to reproduce material in this unit:

Figures

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